Development of the lunar gravity field model GrazLGM300a




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Abstract

In this contribution we present the latest activities (methods and results) at the Space Research Institute of the Austrian Academy of Sciences for the determination of the gravity field of the Moon, starting from the GrazLGM200a model, which has been published in early 2014. Our research is based on high-precision inter-satellite Ka-band ranging (KBR) observations collected by the Gravity Recovery And Interior Laboratory (GRAIL) mission during the primary mission phase (March 1 to May 29, 2012). We exploit the ranging measurements by an integral equation approach using short orbital arcs. The basic idea of the technique is to reformulate Newton‘s equation of motion as a boundary value problem. This method has already been successfully applied for the recovery of the Earth‘s gravity field from data provided by the Gravity Recovery And Climate Experiment (GRACE). For the development of our new Graz Lunar Gravity Model, GrazLGM300a, we refined modeling and parameterization. We validate our results with the GL660B solution - a recent GRAIL model computed at NASA-JPL, which is also based on observations from the primary mission phase. We show that the actual solution GrazLGM300a represents a distinctive improvement compared to the predecessor model and is close to the models developed at NASA, apart from the spectral resolution.

Keywords: Satellite Geodesy, Gravity, Moon, GRAIL

Kurzfassung

Ausgehend von der Schwerefeldlösung GrazLGM200a, die Anfang des Jahres 2014 publiziert wurde, werden in diesem Beitrag die aktuellen Forschungsergebnisse hinsichtlich der Schwerefeldbestimmung des Mondes am Institut für Weltraumforschung der Österreichischen Akademie der Wissenschaften präsentiert. Die Untersuchungen basieren auf hoch präzisen Ka-Band Distanzmessungen der Gravity Recovery And Interior Laboratory (GRAIL) Mission während der ersten Messphase (1. März bis 29. Mai, 2012). Die Messungen werden anhand eines Integralgleichungsansatzes unter Verwendung kurzer Bahnbögen analysiert. Die grundlegende Idee dahinter ist eine Umformulierung der Newton‘schen Bewegungsgleichung als Randwertproblem. Diese Methode wurde bereits erfolgreich zur Schwerefeldbestimmung der Erde im Zuge der Gravity Recovery And Climate Experiment (GRACE) Mission verwendet. Für die Erstellung des aktuellen Mondschwerefeldes GrazLGM300a wurden Modellierung und Parametrisierung überarbeitet. Die Lösung wird mit dem NASA-JPL Modell GL660B, welches ebenfalls auf Beobachtungen während der ersten Messphase beruht, verglichen. Die aktuelle Lösung GrazLGM300a stellt eine deutliche Verbesserung zum Ausgangsmodell dar und entspricht, bis auf die spektrale Auflösung, annähernd den NASA Modellen.

Schlüsselwörter: Satellitengeodäsie, Schwerefeld, Mond, GRAIL

1. Introduction

Since the beginning of the space age in the late 1950‘s, the satellite-based exploration of the Moon is an important subject in scientific research. As a result, nearly 100 lunar missions with a wide range of science objectives were launched (more or less successfully) between 1959 and 1976.

Concerning lunar gravity, a milestone was the discovery of mass concentrations, so called mascons [16], based on data from the Lunar Orbiter V mission (1968). A major improvement in lunar gravity field mapping was achieved with the Lunar Prospector (LP) orbiter, launched in 1998 [1]. Due to the low average altitude of the mission tracking data to the LP spacecraft provided the first lunar gravity field from a low-altitude polar orbiter; based on these findings, research on the tidal deformation and thermal evolution of the Moon as well as the composition, state and size of the lunar core experienced considerable advance (e.g. [7], [8]).

Typically, the determination of the lunar gravity field is accomplished by analyzing orbit perturbations from Doppler shift observations (S-band and X-band) between a Moon-orbiting spacecraft and Earth-bound stations (e.g. the Deep Space Network, DSN or the Universal

Space Network, USN). However, due to the 1:1 spin-orbit resonance of the Earth-Moon system, tracking observations are restricted to the lunar nearside. As a consequence, the incorporation of constraints on the solution, for instance in terms of spectral-domain regularization is necessary [3]. The first satellite mission, which also provided observations over the farside of the Moon, was the Japanese Selenological and Engineering Explorer (SELENE) launched in 2007. The sophisticated mission design incorporated three satellites: a main orbiter in a circular orbit and two sub-satellites in elliptical orbits. In addition to classical (radiometric) tracking data, four-way Doppler tracking between the main orbiter and a relay satellite was employed as well as Very Long Baseline Interferometry (VLBI) measurements between the two sub-satellites and two ground stations on Earth resulting in a considerable improvement of the farside lunar gravity field [4], [17]. Nevertheless, observation accuracy still remained a limiting factor.

The lunar science mission which eclipses any previous attempts to recover detailed gravitational features of the Moon is the Gravity Recovery And Interior Laboratory (GRAIL) mission [20] launched by NASA on September 10, 2011. The mission concept is inherited from the Gravity Recovery And Climate Experiment (GRACE) project, a space gravimetry mission mapping the Earth's gravity field [19]. It consists of two identical spacecraft following each other in a low-altitude near-polar orbit. Table 1 gives an overview of the different mission phases and the orbit characteristics.

For our present gravity field investigations we focus on data solely from the primary mission.

Mission phases	
Launch	September 10, 2011
Primary mission	March 1 to May 29, 2012
Extended mission	August 30 to December 14, 2012
Decommissioning	December 17, 2012
Orbit characteristics	
Altitude	~55 km (±35 km)
Inclination	~89.9° (w.r.t. lunar equator)
Revolution period	113 min
Separation distance	82–218 km
Mean velocity	1.65 km/s

Tab. 1: GRAIL mission phases and orbit characteristics

The primary science instrument aboard each wGRAIL spacecraft is the Lunar Gravity Ranging System (LGRS; [6]), operating in the Ka-Band frequency (Ka-Band Ranging, KBR). Owing to these high-precision inter-satellite observations (about 0.03 μm/s) with global coverage, the twin-satellite mission allows to infer the lunar gravity field with unprecedented accuracy and spatial resolution [10], [12].

This contribution summarizes the processing strategies used for the development of GrazLGM300a – the latest lunar gravity field model computed in Graz. The work represents an extension to the achievements by [5], i.e. improvements over the predecessor model GrazLGM200a. We compare our results with gravity field models from the pre-GRAIL era as well as to recent GRAIL models computed at NASA-GSFC (Goddard Space Flight Center) and NASA-JPL (Jet Propulsion Laboratory).

2. Method and Parametrization

As an alternative to the GRAIL gravity field solutions from NASA, which are based on the solution of the equation of motion via variational equations (e.g., [15]), we propose a short-arc integral equation approach. The basic idea behind the integral equation approach is to reformulate Newton's equations of motion in the inertial space,

$$\ddot{r}(t) = g(t) + a(t) = g(t) + a_b(t) + a_t(t) + a_n(t) + a_r(t), \quad (1)$$

as a boundary value problem [13]. This method has already been successfully applied to the recovery of the Earth's gravity field from data provided by the GRACE mission [14] and is implemented within the GROOPS (Gravity Recovery Object-Oriented Programming System) software package.

The total acceleration $\ddot{r}(t)$ acting on the GRAIL spacecraft can be split into the Moon's gravitational attraction on the satellite $g(t)$ and additional perturbing forces $a(t)$, including third-body accelerations $a_b(t)$, accelerations due to solid Moon tides $a_t(t)$, non-gravitational accelerations $a_n(t)$ and relativistic effects $a_r(t)$. For the compilation of GrazLGM200a [5], solely gravitational perturbations – i.e. a_b and a_t were modelled. Our successor model GrazLGM300a also considers non-gravitational accelerations a_n in terms of solar radiation pressure and relativistic accelerations (Schwarzschild). The latter is in the order of 1E-10 ms^{-2} and determined following the IERS Conventions 2010 [18].

In order to evaluate the influence of solar radiation pressure we use a 28-plate macro-model of the GRAIL spacecraft published by Fahnenstock [2]. Thus, it is possible to calculate the acceleration for each plate separately and sum up the individual contributions:

$$a_{srp} = \frac{\Phi r_p^2 \upsilon}{m_{s/c} c} \sum_{i=1}^{28} A_i \cos\varphi_i \Big[\hat{r}\left(1 - C_{s_i}\right) - \\ - \hat{n}_i \left(\frac{2}{3}\left(C_{d_i} + \alpha_i \varepsilon_i\right) + 2\cos\varphi_i C_{s_i} \right) \Big] \quad (2)$$

In the equation above, $\Phi = 1367\,Wm^{-2}$ denotes the solar flux constant at a distance of 1 astronomical unit, r_p represents the scaling factor to adjust Φ to the actual distance satellite-Sun. Lunar shadowing is solved purely geometrically from the angular separation and diameters of the Earth and Moon [15] with the associated shadowing function υ. The mass of the spacecraft is indicated by $m_{s/c}$, c is the velocity of light, A_i the individual plate area, $\hat{r}$ the direction from the Sun to the satellite, $\hat{n}_i$ the particular unit plate normal (outward directed), φ is the angle between $\hat{r}$ and $\hat{n}_i$, and C_{si}, C_{di}, α_i and ε_i describe the plate specific specular and diffuse reflectivity, the absorptivity and the emissivity, respectively. It has to be kept in mind that only plates which are illuminated by the Sun must contribute to the final solution.

The plot in Figure 1 illustrates the calculated acceleration over the primary mission phase (March 1 to May 29, 2012). Due to the fact that in April 2012 the solar beta angle was always close to 90 degrees, the spacecraft was permanently illuminated by the Sun and thus not shaded by the Moon at any time. At the beginning and the end of the primary mission phase the situation is different. During these times the spacecraft was shaded by the Moon once per revolution (Figure 1 bottom) – implying that the accelerations due to solar radiation pressure to become occasionally zero.

An overview of processing details and standards used for the compilation of the GrazLGM200a and GrazLGM300a models are given in Table 2.

For the computation of the GrazLGM300a model we used release 04 satellite ephemeris and KBR-observations. This has the advantage that time tagging inconsistencies, which were present in the previous releases, are already removed. Nevertheless, for our latest solution we co-estimated a time bias for the KBR observations (1E-04 to 1E-05 seconds) and included the estimation of empirical parameters (1 cycle per revolution and degree-2 polynomials). In case of thruster events the respective satellite observations were rejected. In order to avoid spectral aliasing, short-wavelength signals (from degree and order (d/o) 301 to 660) were reduced from the KBR data using the GL660B model [9]. A better approach would be to determine the gravity field beyond d/o 300; however, this requires enormous computational resources which are presently not available for the authors.

3. Results

In order to assess the quality of different lunar gravity field models we determined degree-error root mean square (DE-RMS) values according to

$$DE - RMS_n = \\ = \sqrt{\frac{1}{2n+1} \sum_{m=0}^{n} \left[\left(C_{nm}^{ref} - C_{nm}\right)^2 - \left(S_{nm}^{ref} - S_{nm}\right)^2 \right]} \quad (3)$$

where n and m are the degree and order, respectively, of the spherical harmonic expansion; C_{nm}^{ref}, S_{nm}^{ref} refer to the reference model GL660B developed by NASA-JPL. Figure 2 shows the performance of various gravity field solutions. The solid black curve indicates the GL660B signal; the orange curve is the difference to the GRGM660PRIM [11] model by NASA-GSFC, which can be considered as the empirical error

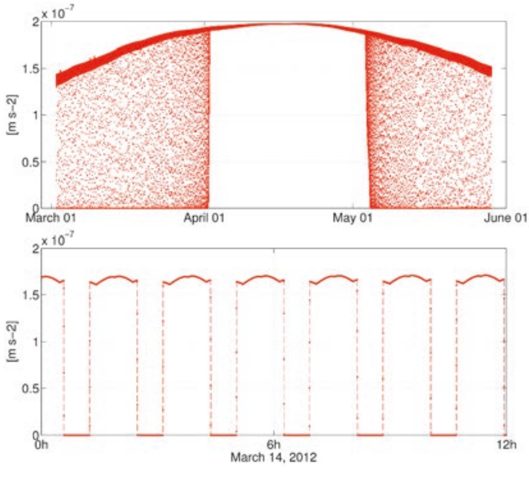

Fig. 1: Accelerations due to solar radiation pressure during the primary mission phase (top) and for half a day in March 2012 (bottom)

	GrazLGM200a	GrazLGM300a
Method	Integral equation approach using short arcs	
Arc length [min]	60	60
Spectral resolution	200	300
Evaluation period	March 2 – May 29, 2012	March 2 – May 29, 2012
A priori gravity field model	JGL165P1	JGL165P1
Planetary ephemeris	JPL DE421	JPL DE421
Spacecraft ephemeris	GNI1B, Release 02	GNI1B, Release 04
KBRR data	KBR1C, Release 02	KBR1C, Release 04
Thruster information	—	THR1B, Release 04
Satellite model	—	28-plate macro-model
Non-gravitational forces	—	Solar radiation pressure
Gravitational forces	Third body accelerations Solid Moon Tides	Third body accelerations Solid Moon Tides Relativistic (Schwarzschild)
Further estimates	—	Time bias Empirical parameters
Outlier handling	Individual weighting of arcs	Individual weighting of arcs
Regularization	—	—

Tab. 2: Standards and models used for the gravity field solutions GrazLGM200a and GrazLGM300a

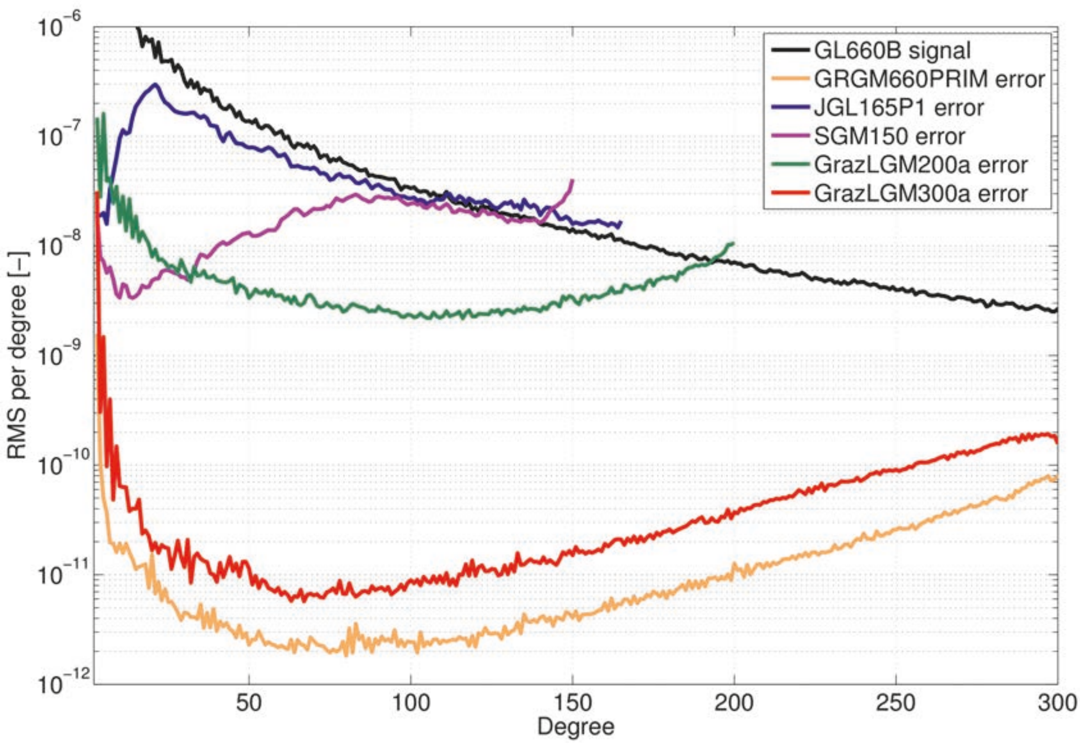

Fig. 2: RMS values per degree. Black solid graph: GL660B signal, other colors: differences to GL660B model.

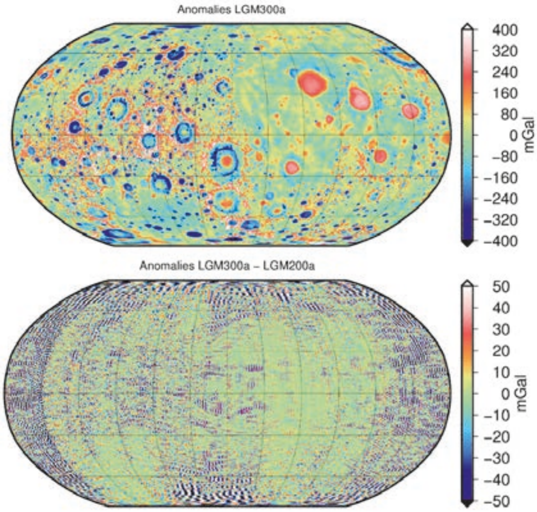

Fig. 3: Free-air gravity field anomalies, evaluated on a spherical 0.5×0.5 grid, top: GrazLGM300a bottom: difference GrazLGM300a-GrazLGM200a. [Projection is centered at 270° eastern longitude; nearside is on the right and the farside on the left of the figure]

of GL660B. Just like our own solutions, both of them are based on data from the primary mission only.

Even though the GrazLGM200a model (green) is based on the worse release 02 data set and considers only gravitational perturbations, it is already superior to the pre-GRAIL models JGL165P1 (LP; blue; [8]) and SGM150 (Selene; magenta; [17]) over almost the entire spectrum. This can be attributed to the high-precision inter-satellite observations of GRAIL and its global coverage. Beside the refined modeling and parametrization, especially the usage of the latest data release led to a distinctive improvement of the GrazLGM300a (red) model compared to GrazLGM200a. Furthermore, due to the correction of KBR data with GL660B (for d/o 301-660) our latest solution remains stable also at higher degrees, whereas GrazLGM200a is getting worse in the short wavelengths. It is gratifying to see that both solutions (GL660B and GrazLGM300a) have nearly the same structure over the whole spectrum and differ just about half an order of magnitude.

In terms of free-air gravity field anomalies the differences between these two models are in the range of ±2mGal, thus Figure 3 (top) shows only results based on the GrazLGM300a model; numerous craters are clearly visible on the nearside and farside of the Moon. In comparison, the GrazLGM200a model shows a much noisier picture on a global scale – mainly originating in the short wavelength range (Figure 3, bottom).

4. Discussion and conclusion

With the availability of high-precision inter-satellite observations by the GRAIL mission, lunar gravity mapping has raised to an unprecedented level in terms of accuracy and spatial resolution. Gravity models based on data from this outstanding mission perform significantly better than earlier models. Therefore, GRAIL gravimetry provides the opportunity to improve our knowledge about the interior structure and thermal evolution of the Moon.

As far as lunar gravity field recovery at the Space Research Institute of the Austrian Academy of Sciences is concerned, we showed that our latest model GrazLGM300a represents a distinctive improvement compared to the predecessor model. Apart from the spectral resolution, the model is close to those developed at NASA over the respective analysis period (primary mission phase). Open issues include the modeling of accelerations due to lunar radiation, the application of a more sophisticated self-shadowing technique, and a refined covariance handling.

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