



## Continuous modelling of point clouds by means of freeform surfaces

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### Abstract

Deformation analysis is one of the classical tasks in engineering geodesy. The development of the laser scanner has changed the respective data acquisition as well as the data analysis: Instead of point based approaches, areal ones move into focus. In this paper a project is presented which aims to develop a spatiotemporal continuous collocation. In order to model the deterministic trend, B-spline surfaces are used. The parameterization required for the estimation of such freeform surfaces is realized by projecting the acquired point cloud onto a based surface called Coons patch. In order to handle irregular point densities, boundary constraints are introduced.

**Keywords:** Laser scanning, modelling, freeform surfaces, boundary constraints, datum definition

### Kurzfassung

Die Deformationsanalyse ist eines der klassischen Aufgabenfelder der Ingenieurgeodäsie. Mit der Entwicklung des Laserscanners haben sich neben der Datenerfassung auch die entsprechenden Auswertestrategien verändert: Anstelle von punktbasierten Ansätzen rücken flächenhafte Methoden immer mehr in den Fokus. Das Ziel des in diesem Paper vorgestellten Projektes ist die Entwicklung einer raumzeitlichen Kollokation, deren deterministischer Trend mit Hilfe von B-Spline-Flächen modelliert wird. Die Schätzung solcher Freiformflächen erfordert eine Parametrisierung der Beobachtungen. Aus diesem Grund wird mit dem sogenannten Coons Patch eine Basisfläche definiert, auf die die erfasste Punktwolke projiziert wird. Für Punktwolken mit unregelmäßigen Punktdichten wird das Ausgleichsergebnis durch das Einführen von Randbedingungen stabilisiert.

**Schlüsselwörter:** Laserscanning, Modellierung, B-Splines, Randbedingungen, Datumsdefinition

### 1. Introduction

The development of terrestrial laser scanners has substantially increased the importance of areal measurements in engineering geodesy [1]. In order to preserve the added value yielded by these techniques in comparison to the conventional point based ones, an areal data analysis is unavoidable. This is in general achieved by extending existing point based approaches.

Least-squares collocation is a well-established method in geodesy. It distinguishes itself from other methods by modelling the observed phenomena not only by means of a deterministic trend, but also by means of a statistical signal.

The aim of the present project is the extension of the classical least-squares collocation to an approach which is continuous in space and time, so that a description of deformations to arbitrary times and in arbitrary places of the object is possible.

The spatiotemporal collocation starts with the modelling of the geometric part of the areal deformation process by means of freeform surfaces such as B-splines. The estimation of this type of surfaces requires the allocation of appropriate

surface parameters to the observations. Setting up the surface's parameter form, is in the focus of this contribution.

B-spline curves and surfaces have been investigated for various geodetic applications: In [2] B-spline surfaces are used as an alternative for spherical harmonics in order to describe the vertical total electron content of the Earth's atmosphere. The observations' parameterization is realised by means of their two-dimensional Cartesian coordinates scaled to the unit square.

In engineering geodesy freeform curves and surfaces are used to model point clouds acquired by laser scanners: The authors of [3] use terrestrial laser scanning in order to determine a bridge's deflection under traffic load. The resulting nonlinear point profiles are parameterized by means of the points' Euclidean distances and afterwards approximated by B-spline curves. The freeform surface's potential to describe deformations is demonstrated in [4]: A plastic sheet is being deformed under pressure while a laser scanner is measuring points on the deforming surface. Three-dimensional B-spline surfaces are used to describe the resulting point clouds analytically. The author assumes a grid-like ar-

ramgement of the measured points which allows a parameterization using the Euclidian distances between the observations.

The present paper is structured as follows: Section 2 provides the mathematical basis concerning the estimation of B-spline curves and surfaces. In section 3 a parameterization approach is introduced, which is based on an object, instead of a superior coordinate system. This guarantees that the parameterization reflects the object's actual form. Furthermore, the presented approach is able to handle unordered point clouds, which generally are a laser scanner's output. As a consequence, no simplifying assumptions about the observations' arrangement have to be made. The achieved parameterization is improved iteratively, whereby irregular point densities are managed by introducing boundary constraints. In the last subsection of section 3 it is discussed whether the parameterization can serve as a basis for the datum definition. The overall results are summarized in section 4.

## 2. Estimation of freeform curves and surfaces

### 2.1 Estimation of B-spline curves

A B-spline curve of degree  $p$  is defined by its  $n + 1$  control points  $P_i$  [5]:

$$C(u) = \sum_{i=0}^n N_{i,p}(u)P_i, \quad u = [0, \dots, 1]. \quad (1)$$

A curve point  $C(u)$  therefore results as the weighted average of the control points  $P_i$ . The corresponding weights are defined by the B-spline basis functions  $N_{i,p}$ , which can be computed recursively by means of the Cox-de Boor-algorithm (see [6]; [7]). In addition to the control points, the degree  $p$  as well as a knot vector  $U = [u_0, \dots, u_m]$  are required to define a B-spline curve uniquely.

When estimating a B-spline curve, the number of control points as well as the curve's degree  $p$  can be specified a priori. This results in a linear relationship between the  $N + 1$  observations  $C(u_k)$  and the unknown parameters  $P_i$ . Before estimating the curve, convenient parameters  $u_k$  have to be allocated to the observations. In case of curve estimation, where the observations are arranged chain-like, either the uniform, the cumulative chord length or the centripetal parameterization is commonly used [8]. In the following the cumulative chord length parameterization is used, whereby the required parameters  $u_k$  are determined by means of the Euclidean distance between neighbouring observations:

$$u_k = \begin{cases} 0, & k = 0 \\ u_{k-1} + \frac{|C(u_k) - C(u_{k-1})|}{\sum_{k=1}^N C(u_k) - C(u_{k-1})}, & k > 0 \\ 1, & k = N. \end{cases} \quad (2)$$

Thus, the observation's parameterization is independent of the curve's spatial orientation.

In order to avoid singularities in the normal equation system, each knot span of the knot vector  $U$  has to contain at least one parameter  $u_k$ . In [5] an algorithm is proposed which guarantees that this requirement is fulfilled. The knot vector  $U$  and its further influence on the curve and surface estimation are not considered here; for further details related to this topic please refer to [5].

### 2.2 Estimation of B-spline surfaces

In order to describe a B-spline surface, the so called tensor product representation is used, which represents a surface as an infinite number of parametric curves running into two different directions. In order to construct such a surface, two one-dimensional basis functions are multiplied [5]:

$$S(u, v) = \sum_{i=0}^m \sum_{j=0}^n N_{i,p}(u)N_{j,q}(v)P_{ij} \quad (3)$$

$$u, v = [0, \dots, 1].$$

A surface point  $S(u_k, v_k)$  is thus given by a grid of  $(m+1) \times (n+1)$  control points  $P_{ij}$ , the degree  $p$  and the knot vector  $U$  in the direction of the parameter  $u$  as well as the degree  $q$  and the knot vector  $V$  in  $v$ -direction.

Similar to the curve estimation, the number of control points  $(m+1$  and  $n+1)$ , the knot vectors  $U$  and  $V$  as well as the surface's degrees  $p$  and  $q$  can be specified a priori. If the measured points were ordered grid-like, the observations could be parameterized by means of the above mentioned methods [8]. In engineering geodesy the grid-like arrangement cannot be presumed; rather, the usually unordered point clouds require an alternative method, which is presented in the following.

## 3. Surface parameterization

A common approach to parameterize unordered point clouds is the definition of a base surface with known parametric form and a subsequent projection of the observed points onto this surface. Via this projection it is possible to allocate

parameters belonging to the base surface's parameter space to the observations. The difficulty of this procedure lies in defining an appropriate base surface, which has to fulfil certain criteria according to [8]: On the one hand the base surface has to be as smooth as possible while being a good approximation of the point cloud. On the other hand an unambiguous projection of the observed points onto the surface has to be possible. As the base surface's parameterization influences the parameterization of the surface to be estimated, the former should furthermore reflect the point cloud's form. Regarding an automated analysis, the base surface's formulation should additionally be generally valid and consequently independent from the acquired data set.

Because of the complexity of the relevant surfaces in engineering geodesy, a completely general valid approach is not possible. For this reason, those surfaces will be classified according to the number of delimiting curves in the following section. The resulting classes form the basis for the parameterization.

### 3.1 Characterization of surfaces

Typically, either simple geometric primitives are used as base surfaces or the base surface is constructed from the point cloud's boundary curves [8]. As the strategy depends on the number of curves delimiting the point cloud, it is obvious to use this number as a classification criterion to categorize surfaces, which are relevant in engineering geodesy, into three classes:

- Closed surfaces are delimited by no boundary curve [9]. In engineering geodesy those surfaces occur rather seldom. In these rare cases

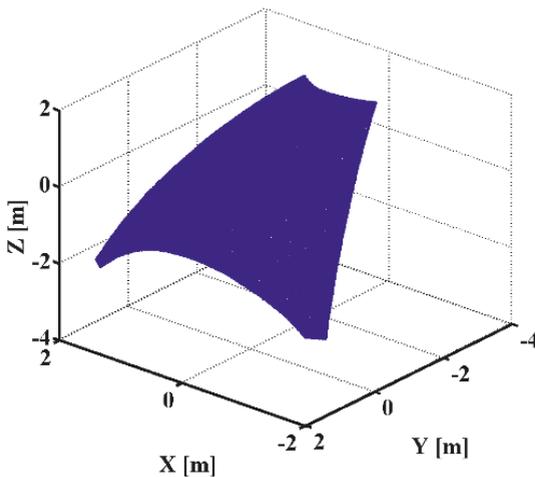


Fig. 1: Point cloud of a church's vault

– for example in case of reference spheres for the terrestrial laser scanning – the surfaces take on simple geometric shapes, so that an unambiguous projection on a sphere is possible. The projection on this base surface is realised by computing spherical coordinates. The final parameterization results from the latitude  $\varphi$  and the longitude  $\theta$  scaled to a range of  $[0, 1]$ .

- Tubes of industrial plants or cooling towers represent surfaces which are delimited by two boundary curves. In engineering geodesy this type of surface also appears in general as a simple geometric shape like a cylinder or a hyperboloid of one sheet. Consequently, these surfaces are parameterized by computing the cylindrical coordinates radius  $r$ , azimuth  $\varphi$  as well as the height  $h$  and scaling  $\varphi$  and  $h$  to the range of  $[0, 1]$ .
- All other surfaces being relevant in engineering geodesy are delimited either by one or by four boundary curves. As a single curve can be subdivided into four segments and a surface delimited by one curve therefore can be transformed into a surface with four boundary curves, these two cases can be handled consistently. These cases are the main focus of this contribution and will be addressed in detail as well as demonstrated on two example data sets, which are presented in the following.

The first data set is a part of a church vault delimited by four boundary curves (see Figure 1). Like most anthropogenic objects, the vault has simple geometric structures and therefore can also be approximated by means of regular geometric shapes.

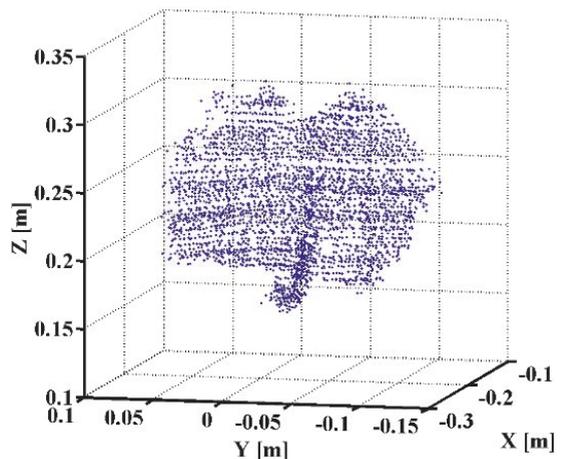


Fig. 2: Point cloud of a leaf

An example for an object delimited by one boundary curve is the leaf of a cucumber plant, which can be seen in Figure 2. Natural objects like this leaf cannot be approximated sufficiently well by means of simple geometric shapes. Furthermore, the point density varies because of the leaf's complex structure, so that the point cloud's approximation by means of freeform surfaces is impeded.

### 3.2 Parameterization of surfaces with four boundary curves

In order to parameterize surfaces which are delimited by four boundary curves, Coons patch (see [8]) is used as a base surface. The first step for the construction of this patch is the determination of four boundary curves in B-spline form. Regardless of whether the surface is delimited by one or by four curves, boundary points have to be detected automatically at first (see [10]). These boundary points form the basis for the estimation of one and four B-spline curves respectively (cf. section 2), which delimit the patch (see Figure 3 and 4). In case only one boundary curve has been estimated, this curve is subdivided into four segments with the same number of control points by means of the Cox-de Boor algorithm afterwards (see [6]; [7]). The results of the curve estimation are four B-spline curves delimiting the point cloud:

$$C_k(u_C) = \sum_{i_C=0}^{n_C} N_{i_C, p_C}(u_C) P_{i_C}^k, \quad k = 0, 1 \quad (4)$$

$$C_l(v_C) = \sum_{j_C=0}^{m_C} N_{j_C, q_C}(v_C) P_{j_C}^l, \quad l = 0, 1. \quad (5)$$

Facing curves are denoted by the letters  $k$  and  $l$  in superscript and have the same degree

as well as the same number of control points. Furthermore, it should be noted that these curves have to be defined on the same knot vector [5]. These conditions have to be fulfilled, as these four curves define the degrees, the number of control points as well as the parameterization of the base surface to be constructed.

In order to distinguish the parameters determining the boundary curves from the parameters of the estimated surface, the former are denoted by the index  $C$ .

Coons patch is constructed by two types of surfaces: On the one hand the two pairs of facing curves are used to construct two ruled surfaces  $R_u(u_C, v_C)$  and  $R_v(u_C, v_C)$  by interpolating linearly between points having the same parameter value [11]:

$$R_u(u_C, v_C) = \left(1 - \frac{i_C}{n_C}\right) P_{0, j_C} + \frac{i_C}{n_C} P_{n_C, j_C}, \quad u_C = \frac{i_C}{n_C} \quad (6)$$

$$R_v(u_C, v_C) = \left(1 - \frac{j_C}{m_C}\right) P_{i_C, 0} + \frac{j_C}{m_C} P_{i_C, m_C}, \quad v_C = \frac{j_C}{m_C}. \quad (7)$$

On the other hand the bilinear interpolant of the four corner points is computed:

$$B(u_C, v_C) = \begin{bmatrix} 1 - \frac{j_C}{m_C} & \frac{j_C}{m_C} \\ \frac{i_C}{n_C} & 1 - \frac{i_C}{n_C} \end{bmatrix} \begin{bmatrix} P_{0,0} & P_{0,n_C} \\ P_{m_C,0} & P_{m_C,n_C} \end{bmatrix} \begin{bmatrix} 1 - \frac{i_C}{n_C} \\ \frac{i_C}{n_C} \end{bmatrix}. \quad (8)$$

The vault's ruled surface in  $u$ -direction as well as its bilinear interpolant are exemplarily shown in the Figures 5 and 6.

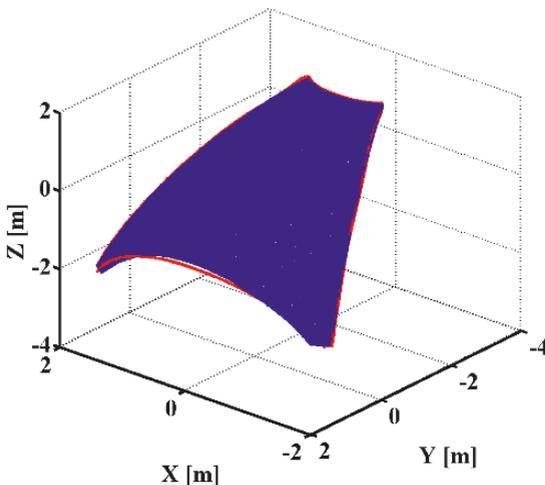


Fig. 3: The vault's four boundary curves

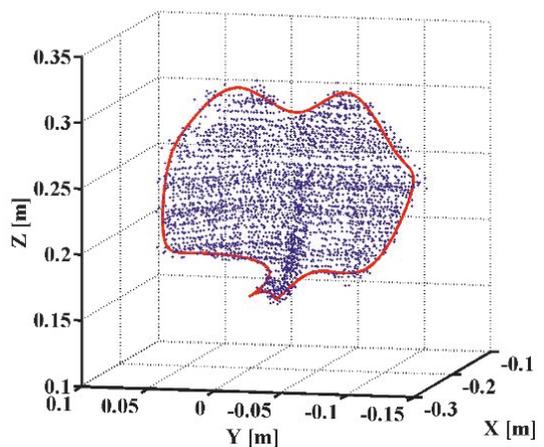


Fig. 4: The leaf's boundary curve

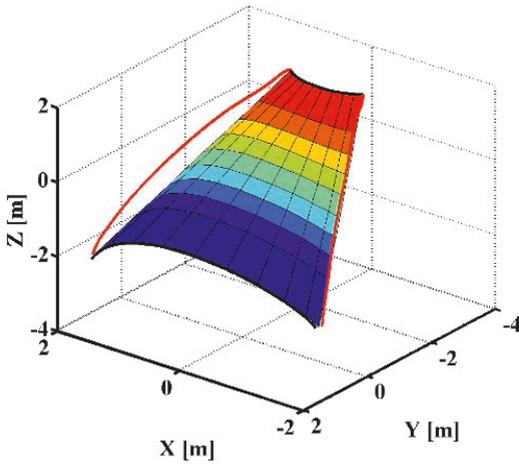


Fig. 5: The vault's ruled surface in  $u$ -direction

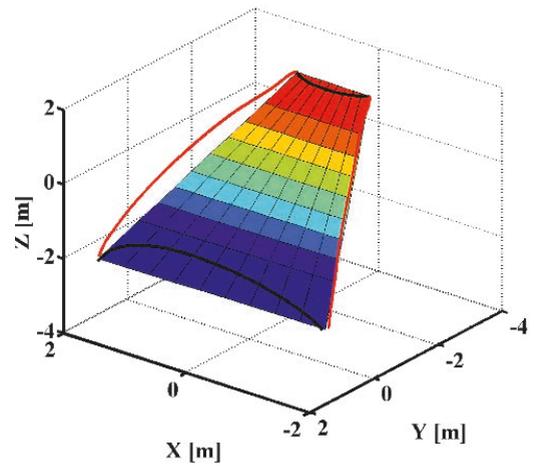


Fig. 6: The bilinear interpolant of the vault's four corner points

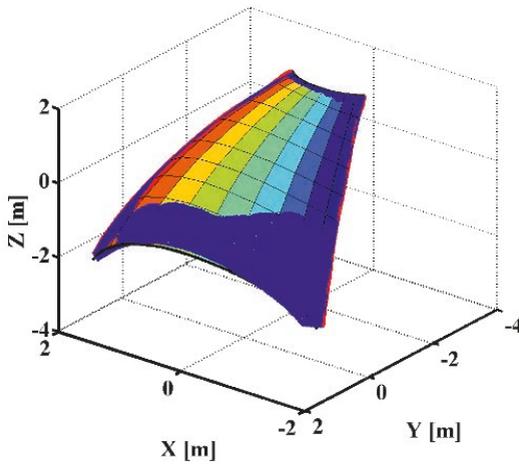


Fig. 7: The vault's Coons patch

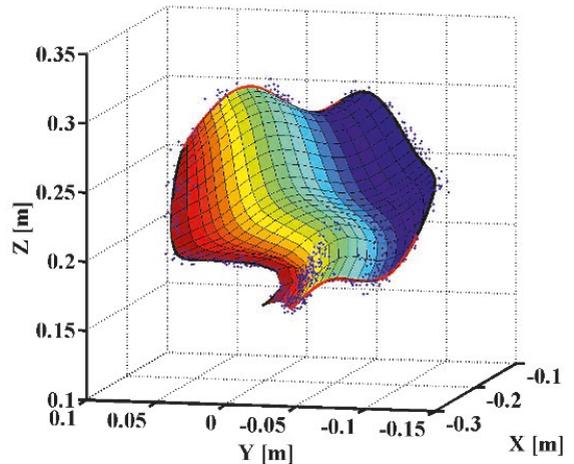


Fig. 8: The leaf's Coons patch

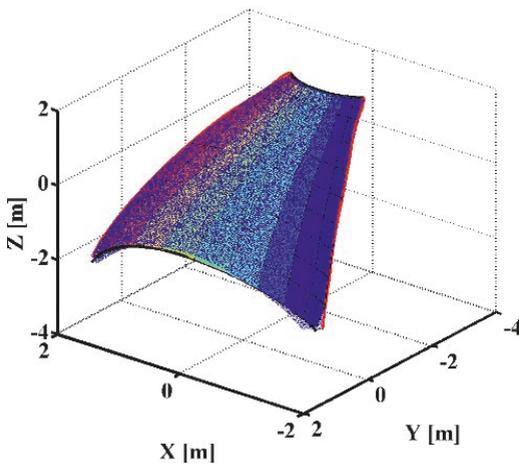


Fig. 9: Estimated vault

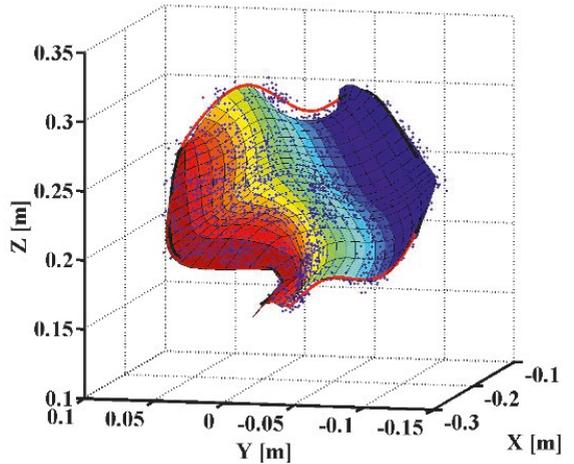


Fig. 10: Estimated leaf

The requested patch  $P(u_C, v_C)$  results as the combination of these three surfaces [12]:

$$P(u_C, v_C) = R_u(u_C, v_C) + R_v(u_C, v_C) - B(u_C, v_C). \quad (9)$$

The vault's and the leaf's patch can be seen in the Figures 7 and 8. Both Figures show that a Coons patch meets the base surface's requirements mentioned above, so that now a base surface exists onto which the observations can be projected to perform the parameterization.

In order to project the points onto the Coons patch, the base surface is subdivided into quadrilaterals which are supposed to be approximately planar (cf. [12]). By means of a principal axis transformation the measured points are transformed into the coordinate system of the nearest quadrilateral, denoted by  $x^q, y^q, z^q$  in the following. The respective  $z^q$ -axis is parallel to the patch's normal vector. As a consequence, the points can be projected to the quadrilateral by discarding that coordinate which contains least information ( $z^q$ -coordinate), leaving the coordinates  $x^q$  and  $y^q$  to each observation. These coordinates form the basis for the determination of the related surface parameters  $u$  and  $v$ : As for each of the quadrilateral's four corner points the respective Cartesian coordinates  $x_j^q$  and  $y_j^q$  ( $j = 0, \dots, 3$ ) as well as associated the parameters  $u_j$  and  $v_j$  are known, the observation's requested parameters  $u$  and  $v$  can be computed by the inverse bilinear interpolation of the quadrilateral's corner points.

### 3.3 Iterative improvement of the parameterization

The parameters determined in this way can now be used to estimate a best-fitting B-spline surface  $S(u, v)$  (cf. section 2). For the setup of the stochastic model the observations are assumed to be equally accurate and uncorrelated. The Figures 9 and 10 show the results of the surface estimation for the vault as well as for the leaf.

The estimated surfaces approximate the point clouds in a better way than the initial base surfaces do. This statement is supported by the numerical values of the point clouds' mean deviations from Coons patch  $\bar{d}_C$  as well as from the estimated surface  $\bar{d}_S$ : The vault's mean deviation reduces from  $\bar{d}_C^V = 0,05\text{m}$  to  $\bar{d}_S^V = 7,5 \cdot 10^{-4}\text{m}$ . Similarly, the leaf's mean deviation decreases from  $\bar{d}_C^L = 0,004\text{m}$  to  $\bar{d}_S^L = -2,4 \cdot 10^{-16}\text{m}$ . The smaller the values, the better the respective surface approximates the point cloud. As a consequence, the estimated surfaces fulfil the base

surfaces' requirements in a better way than the Coons patch does. Thus, a reparameterization based on the estimated surface gives reasons to expect a better result of the adjustment.

#### 3.3.1 Reparameterization

Taking the requirement into account that the base surface's parameterization should reflect the shape of the surface to be estimated as well as possible, at first a new parameter space is defined. The basis of this new parameter space is formed by isolines on the estimated surface each one being reparameterized by means of the cumulative chord length method (cf. equation (2)). Based on this new parameter space, it is possible to allocate improved parameters to the observations. In this way the parameterization is improved iteratively until the process converges. The whole parameterization procedure is summarized in Figure 11 schematically.

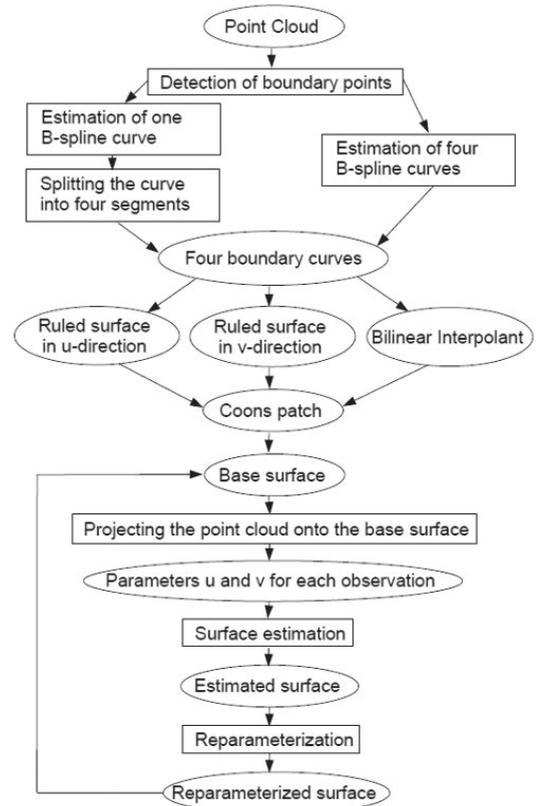


Fig. 11: Parameterization procedure

#### 3.3.2 Introduction of constraints

The iterative adaption of the parameter values leads to artefacts in edge regions with low point densities after just a few iterations (see Figure

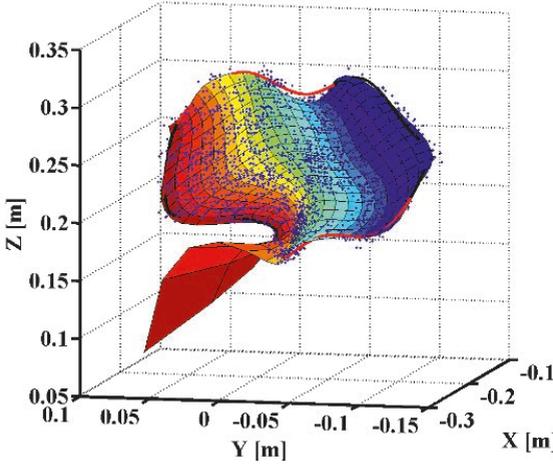


Fig. 12: Artefact in the leaf's edge area (please note the changed range of the z-axis)

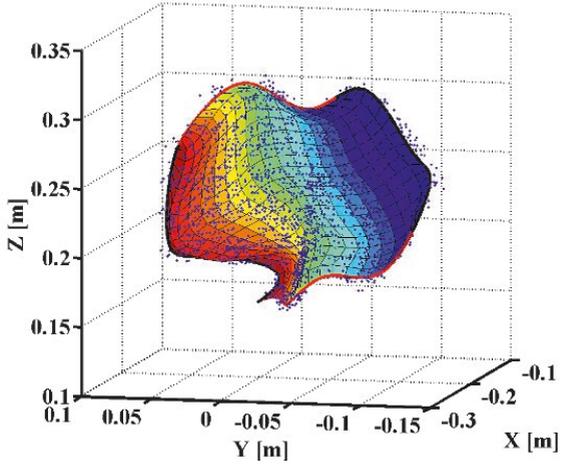


Fig. 13: Constrained estimation of the leaf

12). Consequently, the parameterization does not converge. In order to counteract the surface's degeneration, the knowledge about the surface's limitation is introduced into the adjustment by forcing the surface's edges onto the point cloud's known boundary curves:

$$S(0, v) = C_0(v) \quad (10)$$

$$S(1, v) = C_1(v) \quad (11)$$

$$S(u, 0) = C_0(u) \quad (12)$$

$$S(u, 1) = C_1(u). \quad (13)$$

As the curve's characteristics are determined by the control polygons and as these polygons can be handled much easier than the continuous curves, the constraints are based on the control polygons: The estimated curve control polygons  $\hat{P}_{iC}^k$  and  $\hat{P}_{jC}^l$  with  $k, l = 0, 1$  are treated as known values in the surface estimation. The aim of the adjustment is the determination of the surface control points  $P_{ij}$ . Regarding the constraints, only the outermost control points  $P_{i0}$ ,  $P_3$ ,  $P_{0j}$  and  $P_{mj}$  are of interest.

To provide a better overview, only the case is considered, where the curves' degrees as well as the number of control points are identical with those of the surface to be estimated ( $n_C = n$ ,  $m_C = m$ ,  $p_C = p$ , and  $q_C = q$ ). In this case the outermost surface control polygons can be aligned with the curve control polygons by means of the following constraints:

$$\hat{P}_{iC}^0 = P_{i0} \quad (14)$$

$$\hat{P}_{jC}^0 = P_{0j} \quad (15)$$

$$\hat{P}_{iC}^1 = P_3 \quad (16)$$

$$\hat{P}_{jC}^1 = P_{nj}. \quad (17)$$

These constraints are introduced in terms of pseudo observations (see [13]), so that the constraints' influence on the adjustment's result can be controlled by the corresponding weights. The constraints' introduction provides the desired result (see Figure 13): The edge areas are stabilized and the adjustment converges already after few iterations.

Comparing the isolines in the Figures 13 and 10, the reparameterization's influence on the estimated surface becomes visible: While in the first iteration step especially the inner isolines are clinched, the isolines' distances are quite uniform after the reparameterization.

### 3.4 Parameterization as a basis for datum definition

The classical deformation analysis relies on geodetic networks, whose points are linked by geodetic measurements [14]. The network's seven datum parameters (translations, orientations and scale factor) are introduced by means of datum points [15].

The areal deformation analysis also requires a common reference framework in order to reveal deformations between several measurement epochs. Therefore, the datum definition is also essential when realising areal approaches.

As the base surface's parameter lines define a coordinate system, it seems natural to build up the datum definition from the parameterization. In this case the coordinate system's origin is specified by the base surface's point  $S(u = 0, v = 0)$ . Two of the required coordinate axis are

defined by the isolines  $u = 0$  and  $v = 0$ , so that a curvilinear coordinate system results. The remaining third axis is specified by the surface normal in the system's origin. A significant difference to classical coordinate systems is the definition of the scale factor: Usually a constant scale factor is defined for each coordinate direction, whereas, as the parameter-lines do not run equidistantly, the scale factor varies all over the surface when the datum definition is based on the parameterization. The scale factor along the third coordinate axis, however, can be chosen equidistant.

Naturally, as the parameterization changes during the iterative reparameterization process, the datum definition changes too. However, as the intermediate results are only used to achieve a final parameterization and consequently have no influence on further computations, the associated datum definitions are irrelevant; only that datum definition which belongs to the final parameterization is of importance.

Regarding deformation analysis, not only the datum definition matters, but also the datum's consistency during several measurement epochs [14]. In order to guarantee this consistency, the final base surface of the first measurement epoch has to act as a base surface for the subsequent measurement epochs. When strong deformations occur, this base surface is no longer a good approximation of the point cloud, so that a deterioration of the parameterization has to be expected. For this reason, the following considerations are made under the justified assumption that in engineering geodesy the deformation is small compared to the object size.

Further problems occur, when the object edges change compared to those of the first measurement epoch during the investigation period. If this change is caused by a mere rigid body motion, a new temporary base surface can be defined for the subsequent measurement epoch. Regarding the control points of the two surfaces as homologous, the rigid body motions' translational and rotational parameters can be determined by a similarity transformation and the point cloud of the subsequent epoch can be transformed. Afterwards, a parameterization on the basis of the base surface defining the datum is possible.

If, however, the object deforms, a datum definition based on the parameterization will not be possible: If the point cloud expands beyond the base surface's boundaries, parameters  $> 1$  and/

or  $< 0$  will occur. If the point cloud contracts, the edge regions will be poorly or even not at all filled, so that a singular normal equation system results.

A datum definition based on the parameterization is thus suitable only for special cases (small deformations compared to the object size and unchanged object boundaries as long as no mere rigid body motion occurs). For the general case, however, the datum definition has to be based on a superior and object-independent coordinate system.

#### 4. Summary

The presented method allows a parameterization of unordered point clouds, which serves as a basis for estimating freeform surfaces. The parameterization's fundamental principle is the definition of an appropriate base surface, onto which the point cloud is projected. Depending on the number of boundary curves, either a sphere, a cylinder or Coons patch is used as a base surface. The parameterization obtained by the projection is improved iteratively, while constraints are used to stabilise the edge regions and to counteract the surface's degeneration.

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