

# GNSS/IMU integration for the precise determination of highly kinematic flight trajectories







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## Abstract

An indispensable prerequisite for operating an airborne laserscanner for point determination on or close to the earth's surface is the knowledge about the precise spatial position and orientation of the laserscanner. These parameters of the aircraft's (respectively scanner) trajectory can be determined using a multi-sensor system which consists of a GNSS receiver and an inertial navigation system. This article focuses on the basic principles of IMU/ GNSS integration and the comparison of a combination software, developed at TU Vienna, with the commercial software Waypoint. Further investigations cover the implementation and modelling of the IMU sensor errors.

Keywords: aircraft trajectory, GNSS, IMU, Kalman-filtering, dead reckoning

#### Kurzfassung

Eine Voraussetzung für die Bestimmung von Punkten auf und nahe der Erdoberfläche unter Verwendung eines luftfahrzeuggestützten Laserscanners ist die Kenntnis der räumlichen Position und der räumlichen Orientierung des Laserscanners während des Fluges. Die Bestimmung dieser Parameter erfolgt aus Messungen eines Multisensorsystems, bestehend aus einem GNSS Empfänger und einem Trägheitsnavigationssystem. Dieser Artikel beinhaltet die Grundprinzipien der IMU/GNSS Integration sowie den Vergleich einer Integrations-Software, entwickelt an der TU Wien, mit der kommerziellen Software Waypoint. Weitere Untersuchungen befassen sich mit der Modellierung und Implementierung der systematischen Fehler der IMU.

Schlüsselwörter: Flugzeugtrajektorie, GNSS (Globales Navigationssatellitensystem), IMU (Inertiale Messeinheit), Kalmanfilterung, Koppelnavigation

#### 1. Motivation

To measure points on or close to the earth's surface by laserscanning, it is necessary to have precise knowledge about the current position and spatial orientation of the scanner. To obtain decimetre accuracy at points on the earth's surface, the parameters of the trajectory must be determined with an accuracy of a few centimetres for the position and a few mgon for the spatial orientation.

This article summarizes the results of the diploma thesis [1], which was carried out at TU Vienna, Institute of Geodesy und Geophysics in collaboration with the Austrian laserscan data provider GeoService. It describes the basics in GNSS/IMU integration, highlights the implemented model and presents first results of a developed combination software.

In the following the design and structure of the developed Kalman-filter algorithm are presented. The filter is tested by using GNSS and IMU measurements of a 2 hours test flight, which has been carried out by GeoService. This flight was performed by means of a helicopter. The helicopter was equipped with a Topcon GNSS receiver and a navigation grade IMU (iNAV-FJI-AIRSURV-001) which is one of the most accurate inertial systems for non-military applications. The GNNS receiver operates with a data rate of 5 Hz. The IMU comprises three coaxially arranged pendulous accelerometers, three optical gyroscopes and operates with a data rate of 1000 Hz.

The results of the test flight are compared with a reference trajectory, which is calculated with the commercial software Waypoint. In comparison to Waypoint the new algorithm is developed in an open and transparent manner. Thus extensions like modelling and estimation of systematic sensor errors can be easily implemented.

## 2. GNSS and IMU processing

There are several options for integrating GNSS and IMU data by Kalman-filtering, whereas the loosely coupled approach is very common. This means the combination is based on the individual results of GNSS and IMU processing.





Fig. 1: Procedure for solving the navigation equations

To reach the aspired accuracy of a few centimetres from GNSS processing, relative kinematic positioning is used. For IMU processing the relation between the measured quantities  $f^b$  (accelerations) and  $\omega_{ib}^{b}$  (angular velocities), accelerations and angular velocities, and desired quantities has to be built. The desired quantities are the position  $r^{e}$ , velocity  $\boldsymbol{v}^{l}$  and spatial orientation  $\boldsymbol{R}_{b}^{l}$  of the sensor at each epoch during the flight. The superscript e denotes the earth-fixed coordinate frame (e.g. ITRF), in which the positions are computed. l labels the local level frame, in which the obtained velocities are orientated. The spatial orientation is described by the attitude matrix  $\boldsymbol{R}_{b}^{l}$ , which represents the rotation between body frame and local level frame. The relation is represented by a set of differential equations (1.a-c), which are called 'navigation equations' [2].

$$\dot{\boldsymbol{r}}^e = \boldsymbol{D}^{-1} \boldsymbol{v}^l \tag{1a}$$

$$\dot{\boldsymbol{v}}^{l} = \dot{\boldsymbol{R}}_{b}^{l} \boldsymbol{f}^{b} - \left( \boldsymbol{\Omega}_{il}^{l} - \boldsymbol{\Omega}_{ie}^{l} 
ight) \boldsymbol{v}^{l} + \boldsymbol{g}^{l}$$
 (1b)

$$\dot{\boldsymbol{R}}_{b}^{l} = \boldsymbol{R}_{b}^{l} \left( \boldsymbol{\Omega}_{ib}^{b} - \boldsymbol{\Omega}_{il}^{b} \right)$$
 (1c)

The basis for the derivation of the navigation equations is Newton's Law, which enables the description of a moving object in inertial space. The matrix D in (1a) performs the transformation between the local frame and the earth fixed frame. For navigation applications on or close to the earth's surface, the measured quantities are superimposed by earth gravity. To obtain the accelerations, which are responsible for the transla-

tion of the helicopter, the measured accelerations need to be corrected by the gravity vector  $g^{l}$  (eq. 1b). The second term of (1b) describes the Coriolis acceleration, which occurs due to the motion of the helicopter relative to the rotating earth.  $\mathbf{\Omega}_{il}^{l}$  is the skew-symmetric form of  $\boldsymbol{\omega}_{il}^{l}$  which is the rotation rate of the local level frame with respect to the inertial frame and  $\Omega_{i_e}^l$  is the skewsymmetric form of  $\omega_{ie}^{l}$  which is the rotation rate of the earth-fixed frame with respect to the inertial frame. Equation (1c) combines the attitude matrix  $\boldsymbol{R}_{b}^{l}$  with the gyro measurements  $\boldsymbol{\Omega}_{b}^{b}$ . As we only need the angular velocities between the local and the body frame, the gyro measurements are compensated by  $\Omega^b_{il}$  (thus, the angular rate between the local level frame with respect to the inertial frame, represented in the body frame). The position, velocity and orientation can be obtained by numerical integration of the measured IMU quantities  $f^b$  and  $\omega_{ib}^b$ . This is known as *free*inertial navigation [2]. According to [3], the procedure is shown in Figure 1.

# **GNSS / IMU integration**

For the integration of GNSS and IMU data a Kalman-filter is used. The Kalman-filtering was developed in the late 50's by Rudolf Kalman. It is especially suitable for the estimation of nonstationary random processes [4]. Besides the measurements the Kalman-filter uses additional information about the time dependent behaviour of the system. This behaviour may be modelled by differential equations. The fundamental relations for many time dependent processes are shown in eq. (2).

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(t)\boldsymbol{x}(t) + \boldsymbol{C}(t)\boldsymbol{\omega}(t)$$
(2a)

$$\boldsymbol{l}(t) = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{v}(t)$$
(2b)

In the system equations (2a)  $\boldsymbol{x}(t)$  is the time dependent state vector.  $\boldsymbol{F}(t)$  is called 'system matrix', which describes the time dependent behaviour. The system error vector  $\boldsymbol{\omega}(t)$  describes the uncertainties of the model referring to reality.  $\boldsymbol{C}(t)$  is the corresponding noise matrix. Using the design matrix  $\boldsymbol{A}(t)$ , the measurement equations (2b) combine the state vector with the measurement vector  $\boldsymbol{l}(t)$ , whereas  $\boldsymbol{v}(t)$  denotes the measurement noise. The quantities  $\boldsymbol{\omega}(t)$  and  $\boldsymbol{v}(t)$  describe Gaussian distributed, white noise processes [2]. The state vector has the following form.

$$\boldsymbol{x}(t) = (\underbrace{\delta\varphi \ \delta\lambda \ \deltah}_{\delta\boldsymbol{r}^e} \underbrace{\delta\boldsymbol{v}_n \ \delta\boldsymbol{v}_e \ \delta\boldsymbol{v}_d}_{\delta\boldsymbol{v}^l} \underbrace{\boldsymbol{\varepsilon}_n \ \boldsymbol{\varepsilon}_e \ \boldsymbol{\varepsilon}_d}_{\boldsymbol{\varepsilon}})$$
(3)

The state vector (3) is typical for the loose coupling strategy, where  $\delta\varphi$ ,  $\delta\lambda$  and  $\delta h$  are the deviations between the IMU's computed and the true position. Furthermore,  $\delta v_n$ ,  $\delta v_e$  and  $\delta v_d$  are the deviations in velocity, orientated in north, east and down direction,  $\varepsilon_n$ ,  $\varepsilon_e$  and  $\varepsilon_d$  are small rotation angles, which describe the deviations in the attitude matrix  $\mathbf{R}_b^l$ .

The system equations of the Kalman-filter are derived from (1) by linearisation using either a Taylor series expansion or perturbation analysis. According to [5], the results of the perturbation analysis are shown in (4.a-c).

$$\delta \dot{\boldsymbol{r}}^e = \boldsymbol{F}_{\dot{r}r} \delta \boldsymbol{r}^e + \boldsymbol{F}_{\dot{r}v} \delta \boldsymbol{v}^l \tag{4a}$$

$$\delta \dot{\boldsymbol{v}}^{l} = \boldsymbol{F}_{\dot{v}r} \delta \boldsymbol{r}^{e} + \boldsymbol{F}_{\dot{v}v} \delta \boldsymbol{v}^{l} + \boldsymbol{\varepsilon}^{l} \times \boldsymbol{f}^{l} + \boldsymbol{R}_{b}^{l} \delta \boldsymbol{f}^{b} + \delta \boldsymbol{g}^{l} \quad (\text{4b})$$

$$\dot{\boldsymbol{\varepsilon}}^{l} = \boldsymbol{F}_{\dot{\varepsilon}r} \delta \boldsymbol{r}^{e} + \boldsymbol{F}_{\dot{\varepsilon}v} \delta \boldsymbol{v}^{l} - \boldsymbol{\omega}_{il}^{l} imes \boldsymbol{\varepsilon}^{l} - \boldsymbol{R}_{b}^{l} \delta \boldsymbol{\omega}^{b}$$
 (4c)

 $F_{ij}$  are submatrices of the system matrix F. The elements of these matrices can be deduced by partial derivation of the Navigation equations with respect to the desired quantities of the flight trajectory. The quantities  $\delta f^b$  and  $\delta \omega^b$  describe the errors of the accelerometer and gyroscope measurements. In the first realisation of the combination tool those errors were not taken into account. Uncertainties in the gravity vector  $g^l$  are represented by  $\delta g^l$ . This quantity is required in case that the deflection of vertical is not explicitly considered. As a consequence, systematic errors are treated as uncertainties. In Figure 2 the differences of the combined GNSS/IMU trajectory to a reference trajectory are shown. The reference trajectory was computed with the commercial software Waypoint. This software also uses a loose coupling strategy for Kalman-filtering but additionally estimates the systematic IMU errors. These errors can be specified as three accelerometer biases and three gyro drifts (see [6]). In Figure 2 the deviations in latitude, longitude and height are plotted over time. The test flight includes rest periods of approximately 10 minutes at the beginning and at the end of the flight. Those periods where used for zero updates.



Fig. 2: Deviations between new Kalman-filter and Waypoint trajectory

The deviations between the new filter and the Waypoint solution are rather small – within a range of a few decimetres only. The main reason for the remaining residuals is that the systematic IMU errors are still neglected in the new Kalmanfilter. Consequently, the further improvement to the 'cm-range' requires the implementation of realistic IMU error models.

Therefore the residual sensor errors are modelled as Gauß-Markov process of first order, which is defined by the following first-order differential equation:

$$\dot{b} = \beta b + \sqrt{2\beta \sigma_b^2} \tag{5}$$

where  $\beta$  is the reciprocal of the process correlation time and  $\sigma$  is the sensor measurement standard deviation. The new extended state vector has the following form:

Where  $fb_x$ ,  $fb_y$  and  $fb_z$  denote the residual bias errors of the accelerometers and  $\omega d_{x_c} \omega d_y$ ,  $\omega d_z$  the residual gyro drift errors. Furthermore the terms  $+\mathbf{R}_b^l \delta f^b$  in equation (4b) and  $-\mathbf{R}_b^l \delta \omega^b$  in (4c) are now taken into account. The estimation of the Gauß-Markov parameters of each accelerometer and gyroscope and further analysis of the sensors are described in [7]. Representative values for the parameters for the accelerometers are  $\beta = 5.3986^* 10^{-4}$  [1/s],  $\sigma_b^2 = 3.82^* 10^{-10}$  [m<sup>2</sup>/s<sup>4</sup>] and for the gyros  $\beta = 3.494^* 10^{-4}$  [1/s],  $\sigma_b^2 = 8.56^* 10^{-18}$  [rad<sup>2</sup>/s<sup>2</sup>].

One important question is how modelling the sensor errors, affects the results of the new Ka-Iman-filter. In Figure 3 the comparison between the new extended Kalman-filter and the Waypoint trajectory is shown.



Fig. 3: Deviations between new extended Kalman-filter and Waypoint trajectory

As expected the deviations between the two trajectories decrease. Now the deviations are within a range of a few centimetres. This result shows very clear, that the consideration of the systematic IMU errors is necessary when cm accuracy must be obtained.

# 3. Conclusions and outlook

A first Kalman-filter approach for the integration of GNSS an IMU data has been established in an open and transparent form. In comparison with the results obtained by the commercial Waypoint solution it can be noticed that Waypoint still shows a little better performance than our new Kalman-Filter approach. Nevertheless, the new algorithm and the Waypoint solution already match within a range of a few centimetres. This is a very promising basis for future investigations, which are already carried within the Project: "Integrierte bordautonome und bodengestützte Georeferenzierung für luftgestützte Multisensorsysteme mit cm-Genauigkeit" which was approved and funded by FFG (Austrian Research Promotion Agency) in December 2009.

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