A detailed analysis of the astrogeodetic geoid solution in the southeast of Austria

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Abstract

A high-precision geoid solution of Austria has been computed from terrestrial gravity field data by Kühtreiber in 2002. A comparison between the gravimetric and astrogeodetic geoid solution revealed regions with large discrepancies, especially in the southeast of Austria. The following paper deals with a thorough investigation on the data used in this area. In several steps additional deflections of the vertical have been predicted using gravity anomalies for simulating possible new observation points. The effects of including new measurements and especially the error estimation of the least squares collocation are analysed. As a result regions with an insufficient distribution of measured deflections of the vertical have been identified. The output of the simulations is used to define the criteria for the selection of additional measurement points of deflections of the vertical. The new observations have been done using the system ICARUS, developed by Dr. Beat Bürki, ETH Zürich. Final investigations verify the effect of the newly measured points. The comparison of the old solution with the solution including additional points indicates that the main reasons for the major discrepancies are the insufficient distribution of measured points in combination with erroneous measurements.

Keywords: Local geoid, astrogeodetic geoid, gravimetric geoid, deflections of the vertical, least squares collocation, ICARUS, Austria

Kurzfassung:

Eine hochgenaue Bestimmung des Geoids von Österreich wurde im Jahr 2002 von Norbert Kühtreiber durchgeführt. Das Geoid wurde mit Hilfe der Kollokation nach kleinsten Quadraten aus einer Kombination von Schwereanomalien und Lotabweichungen bestimmt. Im Rahmen dieser Berechnungen wurden auch ein rein gravimetrisches und ein rein astrogeodätisches Geoid von Österreich bestimmt. Bei dem Vergleich der beiden Lösungen zeigen sich in einigen Regionen große Differenzen. Die größten Abweichungen treten im Südosten Österreichs auf. Im Rahmen dieser Arbeit wurden diese Abweichungen näher untersucht. In mehreren Simulationen basierend auf der Kollokation nach kleinsten Quadraten wurden verschiedenen Konfigurationen der Lotabweichungspunkte untersucht. Die Ergebnisse dieser Simulationen bildeten die Basis für die Auswahl von Punkten für die Neumessungen der beiden Komponenten. Die Beobachtung der astronomischen Länge und Breite zur Bestimmung der Lotabweichungskomponenten erfolgte mit dem Messsystem ICARUS, welches von Dr. Beat Bürki, ETH-Zürich, entwickelt wurde. Abschließend wurde der Einfluss der Neumessungen auf die Geoidlösung untersucht. Der Vergleich von ursprünglicher und neuer Lösung bestätigt die Annahme, dass eine ungünstige Konfiguration der Lotabweichungspunkte sowie fehlerhafte Messungen für die großen Differenzen verantwortlich sind.

Schlüsselwörter: lokales Geoid, astrogeodätisches Geoid, gravimetrisches Geoid, Lotabweichungen, Kollokation nach kleinsten Quadraten, ICARUS, Österreich

1. Introduction

In 2002 a high precision Austrian geoid has been computed by a combination of deflections of the vertical and gravity anomalies using least squares collocation ([7], [8]). In the context of these investigations, also a pure gravimetric and a pure astrogeodetic geoid has been determined. The comparison of the astrogeodetic with the gravimetric geoid solution reveals regions with large discrepancies (Fig. 1). One reason might be the sparse distribution of the deflections of the vertical in combination with the complex geology in this region; furthermore erroneous measurements and unknown trend components might be reflected by these discrepancies as well. In [7] Kühtreiber assumes that the sparse distribution of deflections of the vertical is mainly responsible for these differences.

In the southeast of Austria, where the biggest discrepancies appear, new investigations have been conducted. The main objectives of these investigations are:

1. Depiction of regions with an insufficient distribution of measured points
2. Identification of erroneous measurements

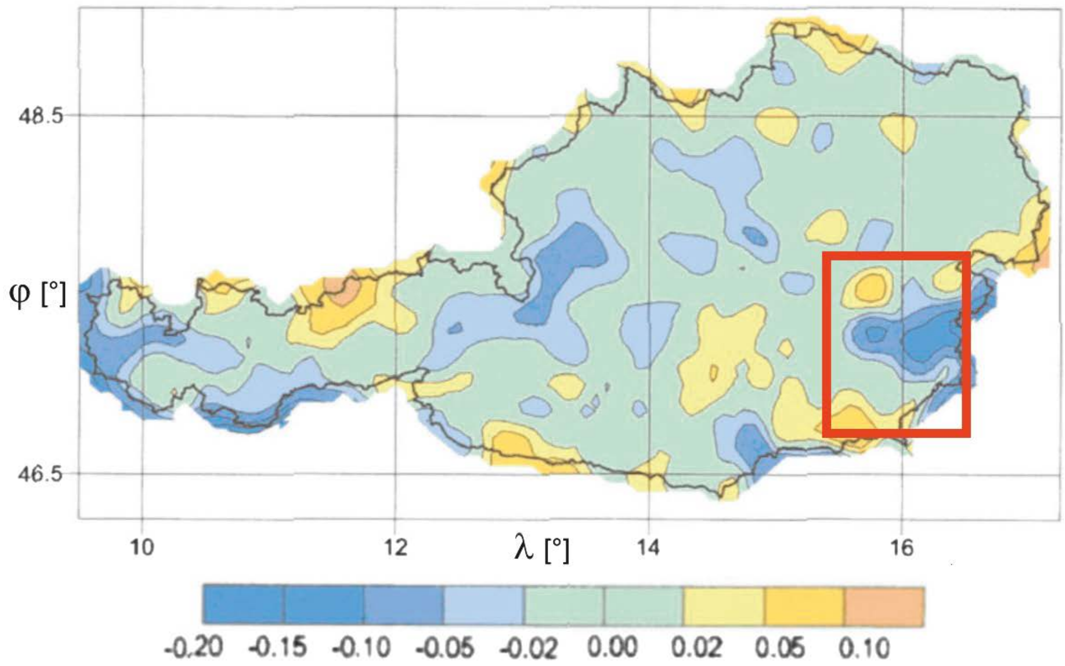

Fig. 1: Gravimetric minus astrogeodetic geoid solution given in cm.

3. Selection of positions for possible new measurements
4. New astronomic measurements for the determination of the deflections of the vertical
5. Analysis of the effect of the new measurements.

The following simulations are based on least squares collocation. The new measurements of the astronomic coordinates are performed with the online observation system ICARUS, which was kindly provided by Dr. Beat Bürki of ETH Zürich, Switzerland.

1.1 Test area

The simulations are concentrated to an area in the southeast of Austria (in the following this area will be called test area). The test area encloses a so called centre zone in which the biggest discrepancies between the pure astrogeodetic and the pure gravimetric geoid solution appear. A red rectangle indentifies this area in Fig. 1. Two aspects have to be taken into account to achieve good conditions for the simulations. First of all the test area has to be expanded beyond the centre zone to reduce possible edge effects. Secondly double points (or points which are very close) causing numerical problems when using least squares collocation and have to be eliminated. The test area is finally chosen from 45.75° to 48.1° N and 14.7° to 17.43° E. The limits of the centre zone are 46.5° to 47.7° N and 15.4° to 16.5° E. The test area includes 1240 points with measured gravity and 192 points with measured deflections of the vertical, where 47 points are inside the centre zone. The 192 measurements of deflections of the vertical used are restricted to the Austrian territory, while the 1240 gravity data points are given in Austria and the neighbouring countries (Hungary and Slovenia).

1.1.1 Input data

Starting in 1978, deflections of the vertical have been determined at 362 stations by the universities of Graz, Vienna and Innsbruck ([9]). At additional 202 stations astrogeodetic measurements have been conducted by the Federal Office of Metrology and Surveying (BEV). Austrian surveying points of first order with a distance of 10 to 15 kilometres have been used for those measurements. The deflections of the vertical have been determined using the Zeiss Ni2 Astrolabium. Additionally Graz University of Technology used a zenith-camera in parallel sessions. The deflections of the vertical refer to the local Austrian datum of the Military Geographic Institute. Later on, the observations have been

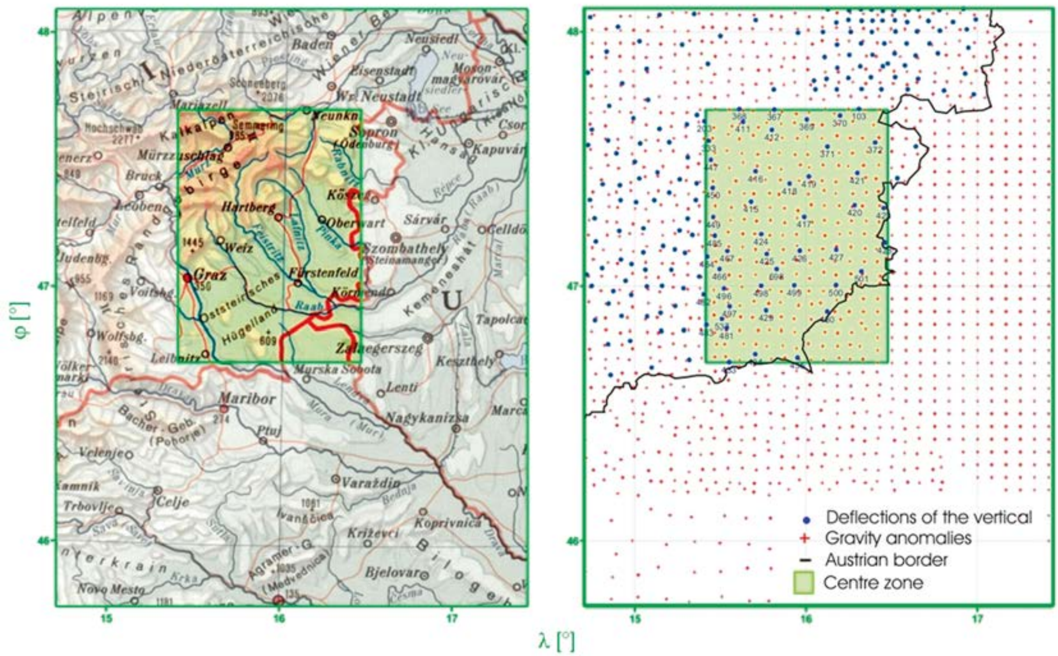

Fig. 2: Test area, Centre zone and data distribution.

transformed to the geocentric system WGS84. Detailed information about the acquisition of data can be found in [9] and in [5].

The Department of Meteorology and Geophysics (University of Vienna), the Institute of Geophysics (University of Leoben), the OMV (Österreichische Mineralölverwaltung Aktiengesellschaft) and the BEV have provided gravity measurements within Austria. At the moment about 86000 gravity observations exist. For the simulations the gravity measurements have been transformed to the geocentric system WGS84. The height system used is the Austrian orthometric height system based on the tide gauge Triest.

The error variances of the gravity anomalies and the deflections of the vertical were determined empirically (see [7]). The error variances were chosen as 1 mgal for the gravity anomalies and 0.2'', 0.3'' for the deflections of the vertical respectively.

In order to remove the long and short wavelength effect of the gravitational potential from the gravity anomalies and the deflections of the vertical two steps are necessary. For the computation of the long wavelength part an adapted EGM96 was used ([1]). For the short to medium wavelengths effect a topographic isostatic reduction was performed using the adapted technique and a detailed height model with the resolution 11.25'' x 18.75''. Detailed Information about the adapted technique can be found in [2]. As the work was based on the investigations by Kühtreiber ([7]) the reduction was done with the height model used in 2002. Meanwhile a more detailed model is of course available from the BEV. Nevertheless investigations done by Kühtreiber showed that the influence of the height model in this area (smooth topography) to the reduction of the deflections of the vertical is not critical (personal correspondence). The used isostatic model was the Airy-Heiskanen model with the standard density of 2.67 g/cm^3, a normal thickness T of 30 km and a crust-mantle density contrast of 0.4 g/cm^3 ([8]).

2. Least squares collocation

2.1 General

The simulations have been done using the well-known Least Squares Collocation approach. A detailed description of the Least squares collocation can be found in [10]. Here, a short summery of the basic equations is given:

$$\hat{s} = C_{sl}(C_{ll} + D)^{-1} l \tag{1}$$

$$C_{\varepsilon\varepsilon} = C_{sl}(C_{ll} + D)^{-1} C_{ls} \tag{2}$$

where l is the vector of the observations, D is the error covariance matrix. The matrix C_{ll} is the covariance matrix of the observations and C_{sl} is the cross-covariance matrix of the observations and the estimated parameters s. $C_{\varepsilon\varepsilon}$ is the error covariance matrix.

The basic covariance function of the disturbing potential is given by:

$$K_{(P,Q)} = \sum_{n=0}^{\infty} \sigma_n(T_P, T_Q) s^{n+1} P_n(\cos\psi) \tag{3}$$

with

$$s = \left(\frac{R^2}{r_P r_Q}\right) \tag{4}$$

where P and Q are the observation points, $\sigma_n(T_P, T_Q)$ denote the degree variances of the disturbing potential, R is the radius of the Bjerhammar sphere, r_P, r_Q are the geocentric radii to the observations P and Q which are separated by a spherical radius ψ and P_n are the Legendre's polynomials.

As the gravity anomalies, the geoid undulation and the deflections of the vertical are linear functionals of the disturbing potential T, the covariance function of these quantities can be derived by covariance propagation ([10]).

2.2 Covariance function of the Austrian gravity anomalies

In order to perform the simulations using least squares collocation an analytical representation of the covariance function of the disturbing potential is necessary. For the model covariance function the analytical expression of Tscherning-Rapp has been used ([12]). The following expressions form the anomaly degree-variance model.

$$\sigma_n(\Delta g, \Delta g) = A\left(\frac{(n-1)}{(n-2)(n+B)}\right) \tag{5}$$

$$\sigma_n(T_P, T_Q) = \frac{R^2}{(n-1)^2}\sigma_n(\Delta g, \Delta g). \tag{6}$$

The Tscherning-Rapp model is characterized by four parameters: A, B, s and N. The three essential parameters of an empirically determined covariance function, the variance C_0, the correlation length ξ and the variance of the horizontal gradient G_0 are used to fit the model covariance function (given by the above four parameters) to the empirical covariance function. The determination of an empirical covariance function and the adapting of the Tscherning-Rapp covariance model parameters already have been done by Kühtreiber in the course of the computation of the Austrian Geoid 2002 ([7]). The computation of the empirical covariance function has been done using all gravity anomalies inside of Austria. The adapting has been done keeping the parameter $B = 24$ fixed. The model parameters A and s have been determined through an iterative adjustment procedure. The computation resulted in the following Tscherning-Rapp degree-variance model parameters: $A = 777.608$ mgal^2; $B = 24$; $s = 0.997002$. In the following simulations the model is used as local covariance function. This means that all degree variances up to a certain degree N are set to zero. The value of N results also from the above described estimation and is equal to 79. Fig. 3 shows the empirical and the adapted model covariance function. All other covariance functions needed are derived from these basic functions using the covariance propagation (see also [10] and [12]). These functions are shown in Fig. 4.

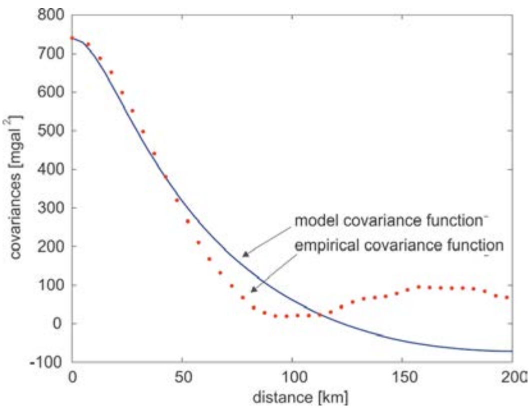

Fig. 3: Empirical covariance function and model covariance function of the gravity anomalies inside of Austria.

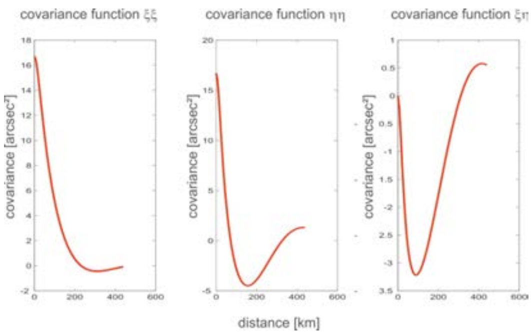

Fig. 4: Covariance functions of the deflections of the vertical.

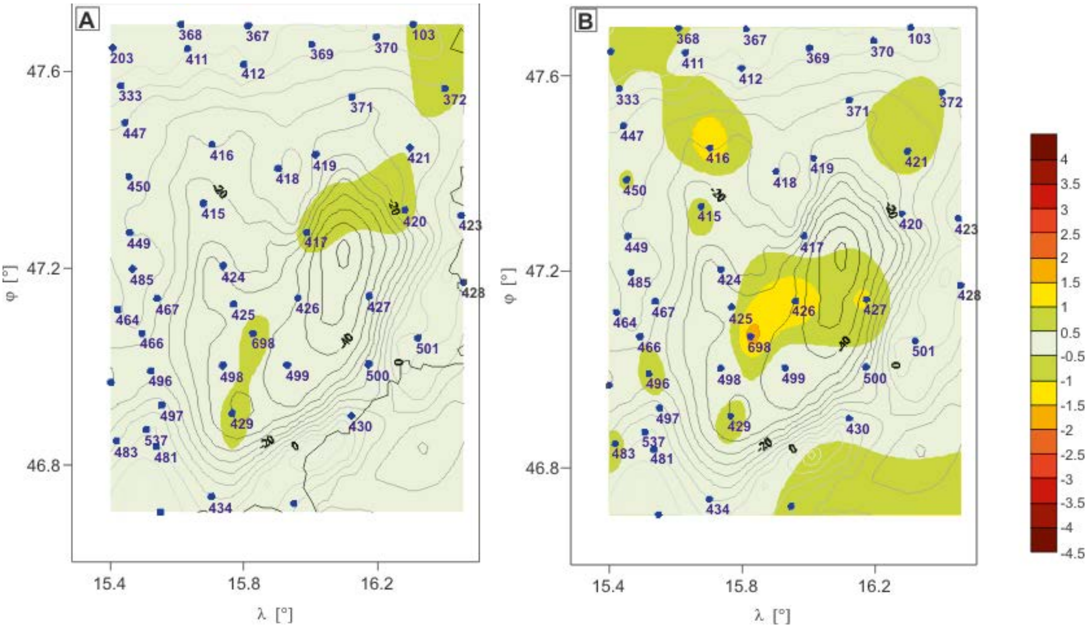

Fig. 5: Differences in arcsec between the predicted and the measured value of ξ (A) and η (B). Prediction using all existing measurements of deflections of the vertical. The black contour lines represent the gravity anomaly field.

The adapted model covariance function has been controlled und verified in following way. The deflections of the vertical are predicted for all 47 points inside the centre zone. As input data all deflections of vertical in the test area are used. The predicted and the measured values are compared to each other. The discrepancies are shown in Fig. 5. There are no significant differences; the adapted model covariance shows a good agreement with the input data.

3. Simulations

3.1 Error estimation by least squares collocation

In a first simulation the error of predicted deflections of the vertical by least squares collocation is estimated. For this purpose deflections of the vertical are predicted on a grid of 4.5' × 4.5'. The input data consists of 192 measured deflections of the vertical. In Fig. 6 the estimated error in arcsec for the predicted deflections of the vertical is shown as colored contour map. The higher errors appear in regions with a sparse distribution of measured deflections of the vertical. If one considers the covariance function und the basic equations of least squares collocation (1) and (2), this result is not very surprising. The covariance function is a function depending on the distance between the input data only. For the error estimation no measurements are necessary and therefore the estimates reflect the configuration of the input data. Thus this test scenario will not be sufficient to identify reasons for the big discrepancies between the pure astrogeodetic and the pure gravimetric geoid solution.

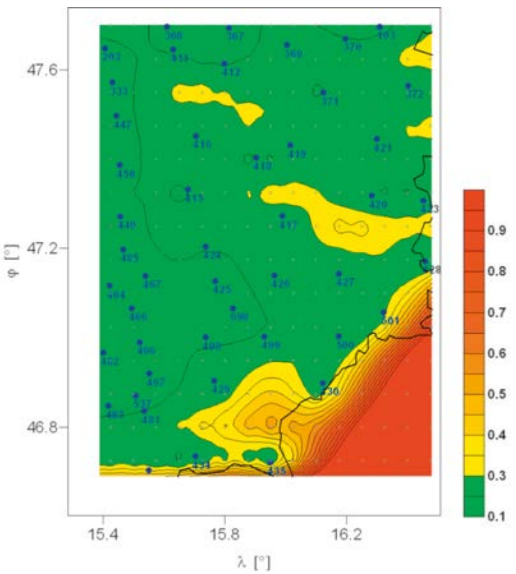

Fig. 6: Error information in arcsec of the prediction on a grid using all deflections of the vertical as input data.

The problem is that for the least squares collocation the error estimation of predicted quantities is a function of the distribution of the measurements and is not correlated to the true error of a measurement. The following thought experiment illustrates this difficulty. Consider a given very smooth gravity field which can be described by a few measurements only. As the gravity field is smooth the true error (expect that the true values for all predicted points are known) will be small. In contrast to that the estimated error of the prediction by least squares collocation will be large as the point distribution is sparse. Vice versa, for a very inhomogeneous gravity field and a dense field of measurements the estimated error will be small while the true error might be large (as still the measurements don't reflect all the features of the gravity field).

3.2 Explanation of the Methodology

As a result of previous considerations (Section 3.1) another simulation scenario has been defined. The simulation is formulated in such a way that also the data values and not only their distribution contribute to the result. In various simulations the measured deflections of the vertical in the centre zone have been predicted either by using deflections of the vertical only (without using the value at the prediction point), or by using gravity measurements only as observations. The predicted values of the deflections of the vertical are then compared to the measured (original) ones. The differences in arcsec for all measurements are shown by colored contour maps.

In a further simulation new deflections of the vertical (possible new points which may be observed) are predicted by using gravity measurements only as input data. Afterwards the predicted values are treated like real measurements and the simulation described above is repeated.

3.3 Validation of the quality of the deflections of the vertical

In the first investigation the quality of the measured deflections of the vertical is evaluated. Details can be found in [14]. The differences between the measured and the predicted deflections of the vertical components ξ and η are shown in Fig. 7. In this case the prediction is done using deflections of the vertical only. In contrast to Fig. 7, Fig. 8 is based on a prediction of deflections of the vertical using gravity anomalies only. As the scaling of the two figures is equal the comparison between the two figures allows the identification of three main features:

1. Big differences are visible in both figures, especially at the measurement points 416, 421 and 698. These measurements seem to be erroneous.

2. The features (values greater than 1.5 arcsec) in Fig. 7A and Fig. 8A may be interpreted as an insufficient data distribution of deflections of the vertical in the northeast. The gravity field is not mapped by the data.

3. Last but not least regions exist where both figures show good results. Of course this is the optimal case, which needs no further investigation.

3.4 Selection of positions for new measurements

As a conclusion of the above investigations it was first of all decided to remeasure the deflections of the vertical at the points 416, 421 and 698. In addition, a densification of the deflections of the vertical is needed. Further simulations were done to identify the number and the position of points for additional measurements, see also [14]. The values ξ and η at these densification points (see triangles in Fig. 9) are predicted by collocation using gravity anomalies only. The improvements we get by introducing these new stations are verified by repeating the above investigation method (cf. section 3.2).

Fig. 9 shows the results of these simulations. If Fig. 7 is compared to Fig. 9 it is obvious that the big discrepancies in the northeast of the centre zone and at the points 416, 421 and 698 have vanished.

4. Measurement campaign

4.1 Measurement system ICARUS

The new measurements are performed using the system ICARUS, an online observation system for rapid and easy determination of the direction of the plumb line in terms of astronomical latitude and longitude. The system also includes a software package which has been developed at the Geodesy and Geodynamics Lab (GGL) at ETH Zurich ([3]). Detailed information about the measurement system ICARUS can be found in [13] and [14].

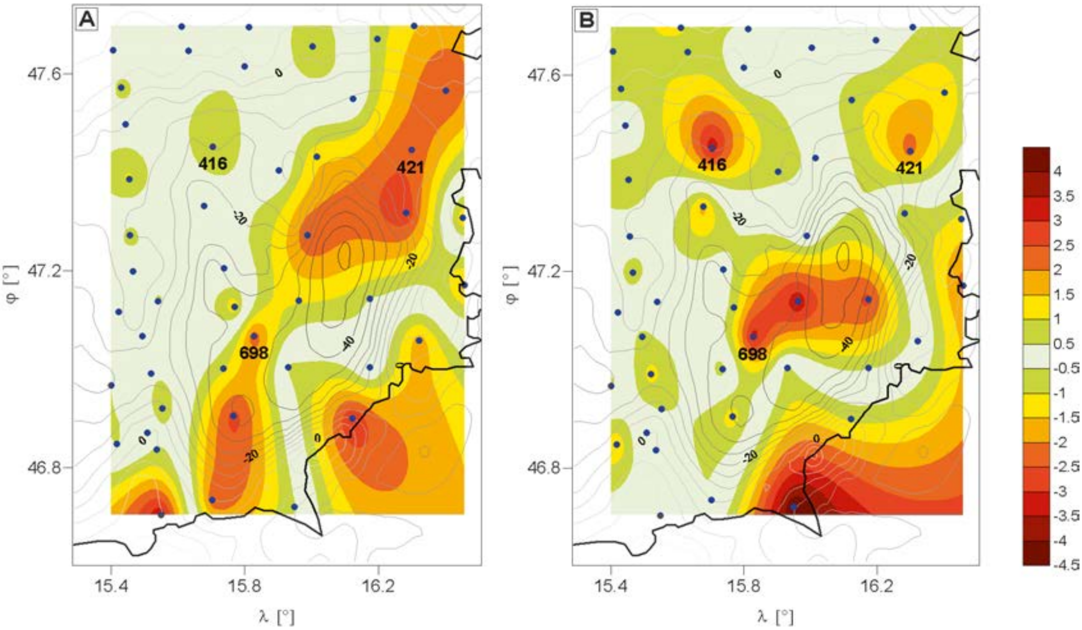

Fig. 7: *Differences in arcsec between the predicted and the measured value of ξ (A) and η (B). Prediction based on existing measurements of deflections of the vertical only. The black contour lines represent the gravity anomaly field.*

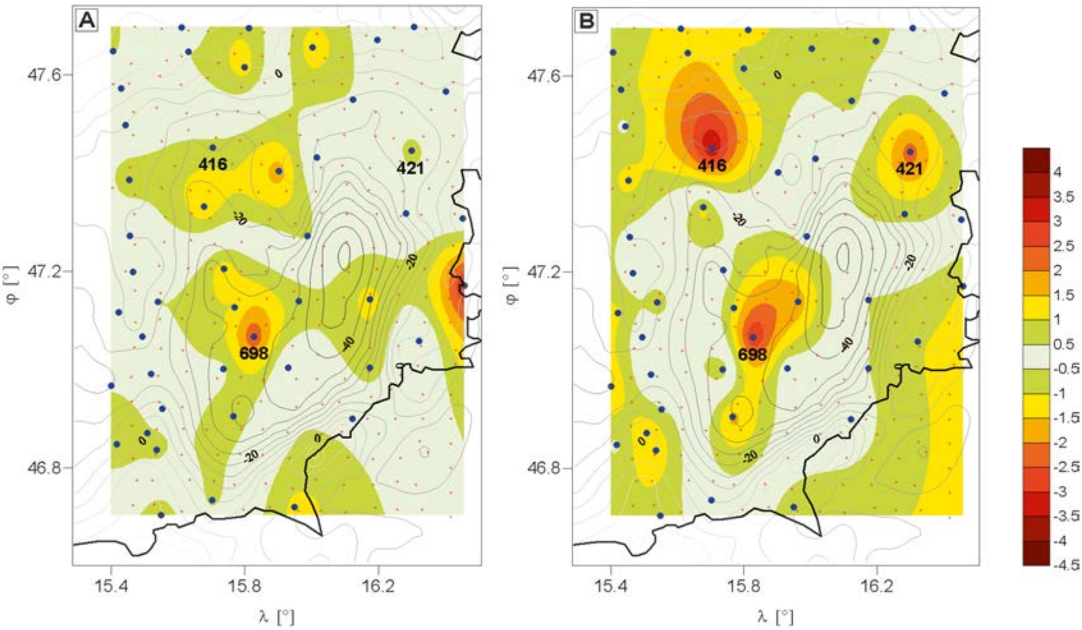

Fig. 8: *Differences in arcsec between the predicted and the measured value of ξ (A) and η (B). Prediction based on gravity anomalies only.*

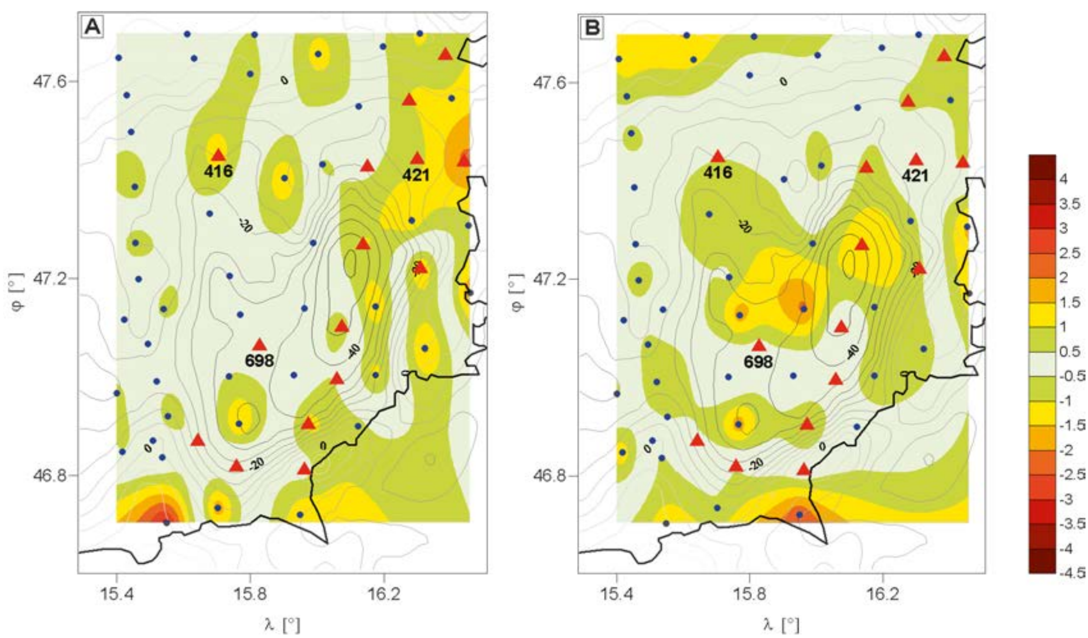

Fig. 9: Improvement by including additional points (marked with a triangle). Differences in arcsec between the predicted and the measured value of ξ (A) and η (B), prediction based on deflections of the vertical only.

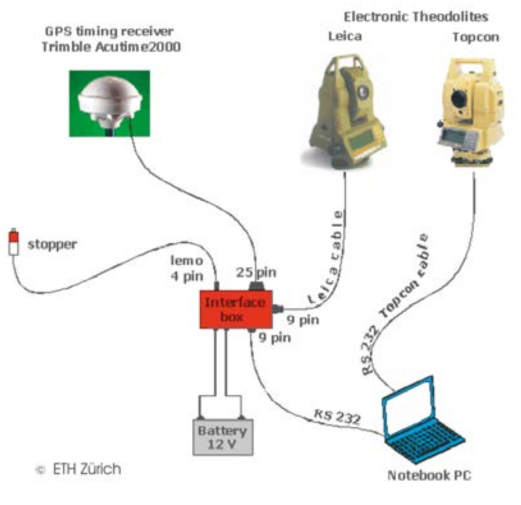

Fig. 10: Components of the measurement system ICARUS.

4.2 Realization

In order to check the whole measurement system (software, GPS receiver and theodolite) a test series of measurements has been done at points with deflections of the vertical known from previous campaigns. At one station (Lustbühel, Graz) the measurement of the deflections of the vertical has been repeated on three different days. The comparison of the measurements shows a root mean square error of approximately $\pm$0.8 arcsec. This value is contradictory to the σ_i of the measurements as given by ICARUS by approximately $\pm$0.2 arcsec. In a second point the differences between existing values of deflections of the vertical and new measured values are less than 0.3 arcsec. These results indicate that the measurements at Lustbühel need a further investigation. In order to get reference values, a remeasurement of the deflections of the vertical could be done by a zenith camera. The zenith camera is the most accurate mobile astrogeodetic observation system available at the moment. Further details about this instrument can be found in [4] and [6].

Based on the results of the simulation described above, 15 points have been selected for new measurements of deflections of the vertical. As observation points (Fig. 11) Austrian surveying points of first order have been used. In a first step the coordinates of the selected Austrian surveying points were validated and the points were identified in the field. The measurements using ICARUS were done in September and October 2006. Due to the distance of two neighbouring stations (about 20 to 40 kilometres) it was possible to measure three or in good circumstances four points during one night. The most time-consuming task at the stations was to set up and initialize the theodolite and the GPS-receiver, and to start the personal computer. Depending on the position of the station (unobstructed view to the sky) and also on the constellation of the stars during the measurement window, the online determination of the astronomical coordinates took about 30-60 minutes.

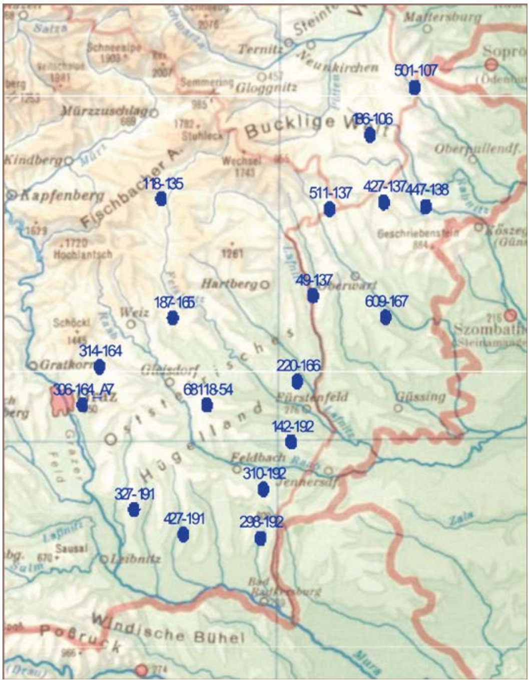

Fig. 11: Austrian surveying points of first order which had been used for the new measurements.

4.3 Data processing

The deflections of the vertical are defined by

$$\xi = \Phi - \varphi \tag{7}$$

$$\eta = (\Lambda - \lambda)\cos\varphi \tag{8}$$

where Φ and Λ are the astronomical coordinates, φ and λ are the geodetic coordinates (see also [10]). After removing the long and short wavelength effect of the gravitational potential (cf. section 1.1) the new measurements have been added to the data set of the deflections of the vertical. In the following an analysis of the new measurements is done by discussing differences. Therefore unless absolute values are needed the restore step can be omitted. Further details and the values of the new determined deflections of the vertical can be found in [13].

5. Analysis of the new measurements

In order to verify the improvements the above described investigations concerning the deflections of the vertical were repeated after including the new data (Fig. 12 to Fig. 14).

5.1 Deflections of the vertical

First of all the comparison of the measured and predicted deflections of the vertical was performed. The prediction was done using either the deflections of the vertical or the gravity anomalies only. In the northeast, due to the densification of the deflections of the vertical, a large part of the differences between the predicted and the measured values of ξ and η has disappeared (Fig. 12).

The results of the prediction using gravity anomalies confirm the assumption that the original measurements of deflections of the vertical at the points 416, 421 and 698 have been erroneous. Comparisons of the new measured deflections of the vertical with the old values see Table 1. After replacing the old measurements with the new ones the discrepancies in these points vanish (Fig. 13). The improvement as a result of the correction of the erroneous measurements is particularly obvious by comparing the deflections component η (Fig. 8B and Fig. 13B).

Deflections of the vertical						
	old ['']		new ['']		differences ['']	
ID	ξ	η	ξ	η	diff ξ	diff η
416	−3.48	5.52	−2.43	3.25	1.05	−2.27
698	−2.63	−1.41	−0.31	2.29	2.32	3.70
421	−1.50	−3.75	−2.51	−1.21	−1.01	2.54

Table 1: New measured deflections of the vertical in comparison with the old values.

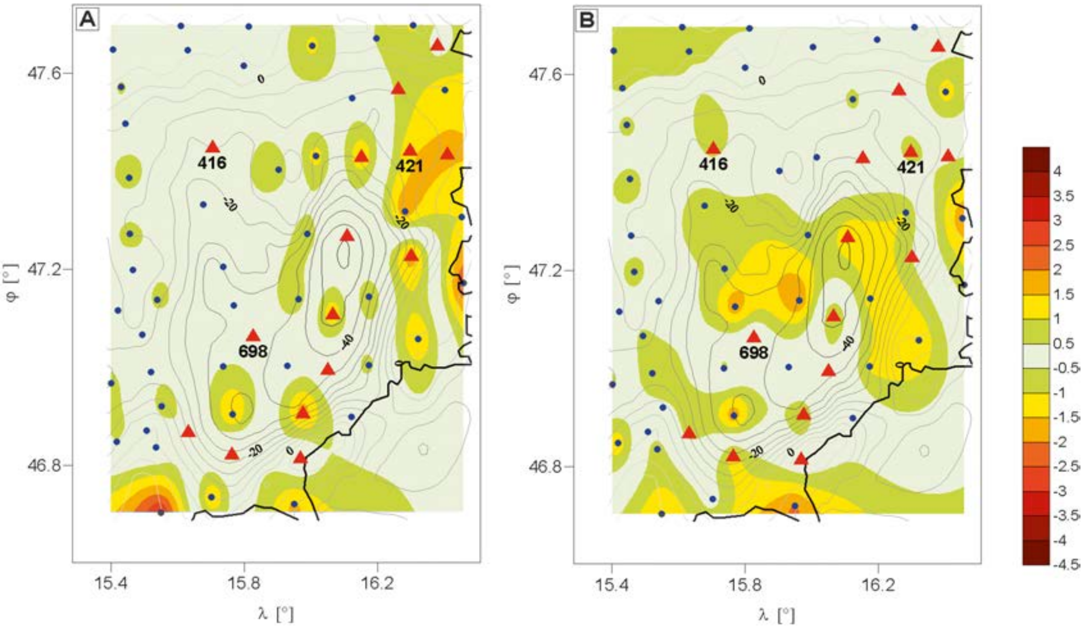

Fig. 12: *Differences in arcsec between the predicted and the measured value of ξ (A) and η (B) including the new measurements. Prediction based on measurements of deflections of the vertical only.*

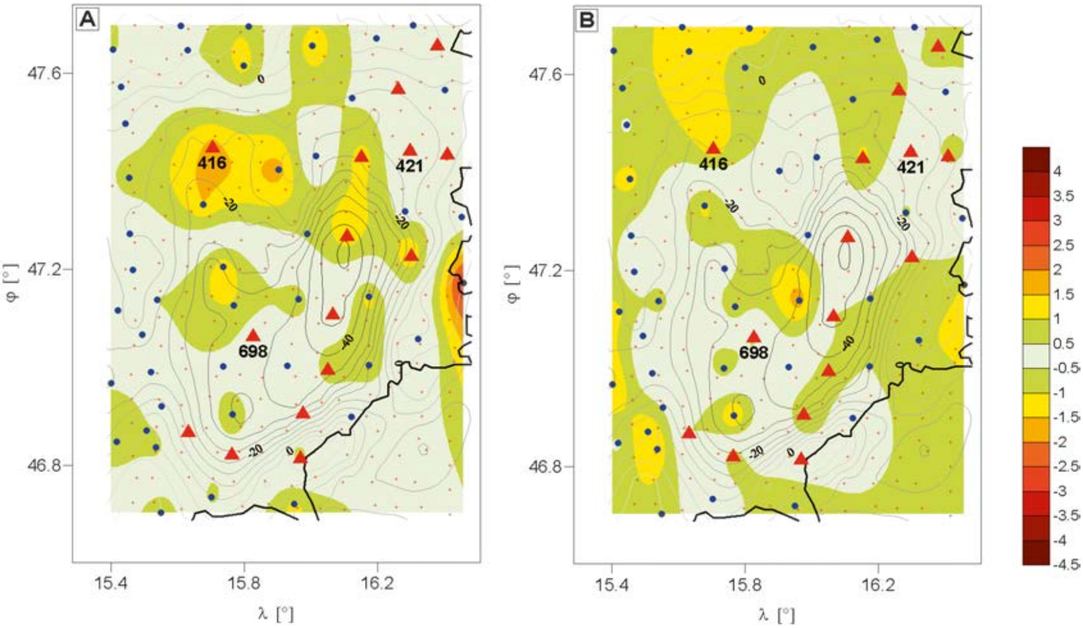

Fig. 13: *Differences in arcsec between the predicted and the measured value of ξ (A) and η (B) including the new measurements. Prediction based on gravity anomalies only.*

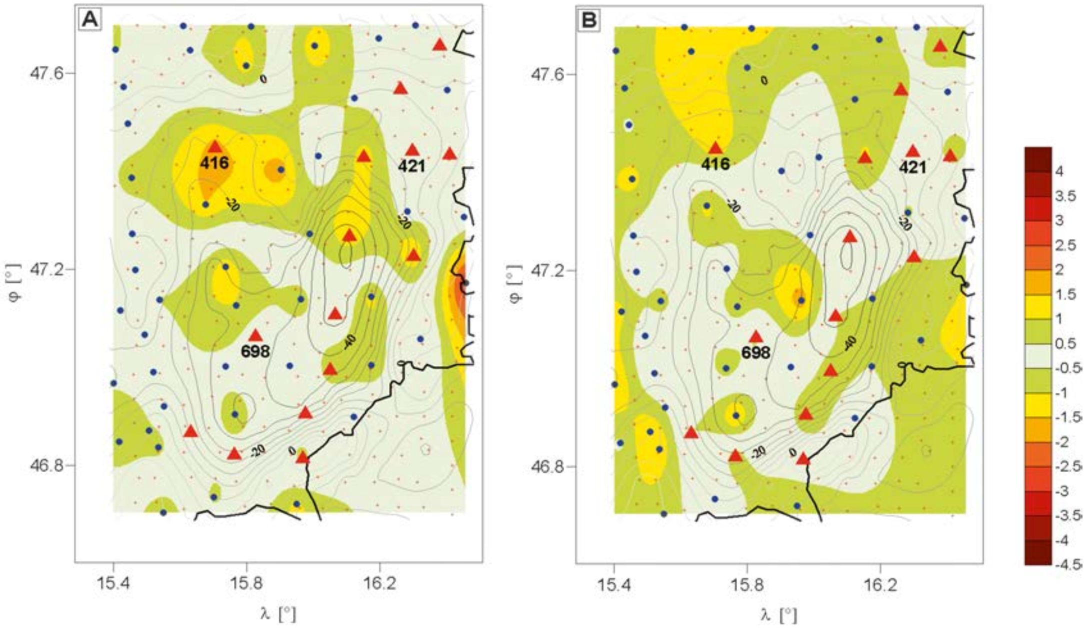

Fig. 14: Differences in meters between the astrogeodetic geoid solutions and the gravimetric geoid; (A) Astrogeodetic geoid based on old dataset of deflection of the vertical, (B) Astrogeodetic geoid based on the new data set.

5.2 Geoid heights

Last but not least, geoid heights have been predicted using least squares collocation. The prediction was done either by using gravity anomalies, the "old" data set of deflections of the vertical or the extended (with the new measurements) data set of deflections of the vertical. The gravimetric geoid differs from the two astrogeodetic geoid solutions mainly by a trend. This trend is caused by insufficient modelling of the long wavelength structures as well as the orientation of the local ellipsoid to the global datum. Before a comparison of the different geoid solutions was done, the trend was approximated by a second order polynomial and removed from the differences. The remaining differences between the gravimetric and the astrogeodetic geoid solutions are shown in Fig. 14. The comparison of the trend reduced geoid height differences confirms the results of the previous investigations. The differences between the astrogeodetic and the gravimetric geoid become significantly smaller after adding the new measurements. One exception is the area in the north of the point 416. Here, the difference between the geoid solutions does not change. There is nearly no improvement. Because the simulations in section 3.3 and 3.4 did not show big differences no new measurements have been planned north of point 416 so far. As the gravity field shows neither a lack of gravity measurements nor a big anomaly Fig. 14 proofs that measurements of the deflections of the vertical are also needed in this region.

6. Conclusions

This investigation in the southeast of Austria shows that the differences between the astrogeodetic and the gravimetric solution are mainly influenced by the distribution of the deflections of the vertical. A densification of the deflections of the vertical instantly leads to better results. Additionally, errors in the data have been detected. The new data set has been included in the determination of the Austrian Geoid 2007, which in detail is described in [11].

Acknowledgments

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