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SLR – Determination of Reflection Time

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Beispiel: Es seien n = 11 Punkte (x_k, y_k) gemäß

x_k	1	3	4	5	6	4	2	0	–1	–2	–1
y_k	–2	–2	0	1	4	5	4	4	2	–1	–3

gegeben. Mit den in Schritt 0 angegebenen Startwerten $t_k^{(0)}$ $(k=1,...,n)$ und $\varphi^{(0)}=4$ wurden für verschiedene Werte von c folgende Resultate erzielt:

c	a	b	q	p	φ	it	S
.5	1.8312	1.1628	-2.5795	-5.1590	-.8101	66	1.48698
1.01	1.6959	1.1457	-3.6919	-3.6554	.5560	≈200	7.80288
1.8782	1.8286	1.1670	-5.0084	-2.6666	.7558	35	1.43124
2.	1.8312	1.1628	-5.1591	-2.5795	.7607	53	1.48696
5.	1.7742	1.1872	-10.134	-2.0268	.7798	≈200	5.6206

In dieser Tabelle bedeutet it die Anzahl der Iterationen, die für 4 Stellen Genauigkeit nach dem Punkt benötigt wurden, und für $c=1.8782$ war S an kleinsten. In Abb. 1 sind die gegebenen Punkte und die resultierenden Ellipsen für $c=1.8782$ und $c=5$ eingezeichnet.

Mit den gleichen Startwerten für $t_k^{(0)}$ aber alternativ mit $\varphi^{(0)}=0$ und $\varphi^{(0)}=.8$ wurden die gleichen Ergebnisse (natürlich andere Werte für it) erhalten, bis auf eine Ausnahme: für $\varphi^{(0)}=.8$ und $c=.5$ wurde ein Nebenminimum mit $S=20.909$ erhalten. Die Ergebnisse für $c=1.01$ deuten an, daß das Problem für $c \rightarrow 1$ schlecht konditioniert ist.

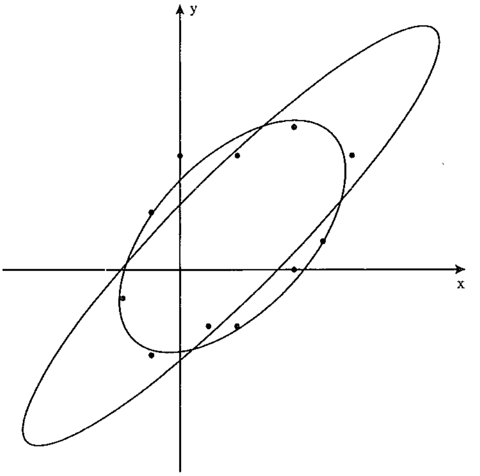

Abb. 1: Gegebenen Punkte und die resultierenden Ellipsen für c = 1.8782 und *c* = 5.

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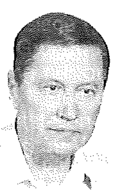

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Josef Kabeláč, Prag

Summary

A new method for the determination of the reflection time of Laser rays from the satellite is submitted. The measured range between observation site and satellite thus will be changed by an estimated maximum of 0.5 mm. The value introduced into adjustments of Satellite Laser Ranging (SLR) should not be the distance but directly the propagation time of light.

Zusammenfassung

Es wird eine neue Methode der Bestimmung der Reflexionszeit von Laserstrahlen von einem Satelliten gegeben. Dadurch wird die gemessene Distanz zwischen Beobachtungsstation und Satellit um ein geschätztes Maximum von 0.5 mm geändert. In die Ausgleichung von Satelliten-Distanzmessungen (SLR) sollte nicht die gemessene Entfernung, sondern direkt die Laufzeit des Lichtes eingeführt werden.

1. Introduction

In the orbital (semidynamic) method of satellite geodesy the distance between the observation site and a satellite is measured. This is called «Satellite Laser Ranging (SLR)». For determining the range the relation is used

$$s = 0.5\, c\, (T_{stop} - T_{start}) + ds, \qquad (1)$$

where c is the velocity of light, $(T_{stop} - T_{start})$ is the travelling time, and ds contains various corrections, like refraction etc. Above relation is valid exactly only in exceptional cases of the mutual positions of the points «site-start», «satellite-reflection», and «site-stop». Therefore it is necessary to introduce a new relation valid for any arbitrary space position of these three points.

2. The new relation for reflection time determination

Figure 1 shows the perturbed satellite orbit and the orbit of the laser site because of the Earth rotation. Point A corresponds to the position of the site at time T_{start}, B at T_{stop}, C and S are the positions of the site and the satellite in the moment of reflection. Δt_1 is the the travelling time A–S and A–C, Δt_2 the travelling time S–B and C–B, with v_1 and v_2 being the speeds of the site, and c the speed of light.

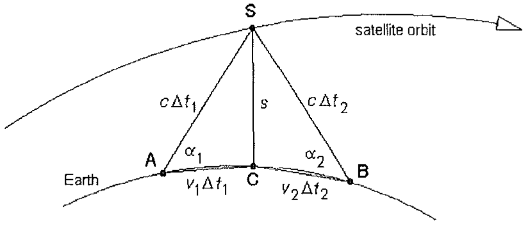

Figure 1: Satellite orbit and the orbit of the laser site because of the Earth rotation

For the travelling times we find the relations

$$\Delta t_j = \frac{s}{c}\left(1 - 2\frac{v_j}{c}\cos\alpha_j + \left(\frac{v_j}{c}\right)^2\right)^{-1}, \text{ for } j = 1, 2. \quad (2)$$

which can be solved by approximation.

First approximation of Δt_1: We determine the coordinates of point A, and by numerical integration (NI) that of the perturbed point $S^{(1)}$. Then with $s^{(1)}$, get $\Delta t_1^{(1)} = s^{(1)}/c$. The input time for the second approximation is thus $T_{start} + \Delta t_1^{(1)}$.

Second approximation of Δt_1: For the improved time $T_{start} + \Delta t_1^{(1)}$ we calculate the coordinates of the point $C^{(2)}$ and again by NI the position of the perturbed point $S^{(2)}$. And further $s^{(2)}$, A–$C^{(2)}$, A–$S^{(2)}$, $\alpha_1^{(2)}$, $v_1^{(2)}$ = (A–$C^{(2)}$)/$\Delta t_1^{(1)}$, and $\Delta t_1^{(2)}$ from Equ. (2). In a similar way the next approximation of is obtained. From the last approximation we get the moment of reflection: $T_{refl.} = T_{start} + \Delta t_1$.

First approximation of Δt_2: $\Delta t_2^{(1)} = s/c$, where s is computed with the last approximation of Δt_1.

Second approximation of Δt_2: For the time $T_{refl.} + \Delta t_2^{(1)}$ we calculate the coordinates of point $B^{(2)}$, further the values of C–$B^{(2)}$, S–$B^{(2)}$, $\alpha_2^{(2)}$, $v_2^{(2)}$ = (C–$B^{(2)}$)/$\Delta t_2^{(1)}$, and $\Delta t_2^{(2)}$ from Equ. (2). The next approximation of Δt_2 follows in a similar way. Finally from the last approximation we get:

$T_{stop} = T_{refl.} + \Delta t_2$.

The same procedure is used for the determination of the influence of the aberration of light.

3. The absolute term

The absolute term of the observation equations of the least squares adjustment is:

$T_{propagation,o} - T_{propagation,c}$,

where

$T_{propagation,c} = \Delta t_1 + \Delta t_2$, and

$T_{propagation,o} = T^*_{propagation,o} + ds/c$,

with $T^*_{propagation,o}$ being the directly measured value, and ds/c the corrections according to Equ. (1).

4. Conclusion

From the time $T_{refl.}$ we can calculate the coordinates of the points S and C and the distance C–S. The difference between this distance and the distance according to Equ. (1) was tested and found to reach a maximum of 0.5 mm.

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