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Austrian Geoid 2000

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result is in agreement with the appropriate specification of line levelling as shown in Table 1 for the NA3003, which have been fully confirmed by all known practical measurements, e.g. [2]. However, the specifications for individual staff readings (Table 1) could not be confirmed by the calibration tests due to the periodic effect present in the true deviations of the NA3000/3 results. These results are summarised in Fig. 10. Therefore the accurate measurement of small height changes as frequently required in industrial applications would be affected by this periodic effect.

The periodic effect of the NA3000/3 equipment which was discovered by the present investigation raises also the fundamental question about the choice of the proper sampling interval for the calibration of a digital levelling system. Shannon's sampling theorem requires the sampling period to be shorter than half the shortest period of the signal which is represented by the true deviations of the height reading in the present case. Fig. 10 also allows to determine the required sampling period using the appropriate periods of the periodic effect as a function of distance.

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Austrian Geoid 2000

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Abstract

By the impact of the relative GPS accuracy of 1 ppm to 0.1 ppm (for longer baselines), the Austrian geoid with its present mean accuracy of about 1 ppm is no longer considered to be sufficiently consistent. For this reason, a new computation of the Austrian geoid was initiated with the objective to obtain a relative accuracy of at least 0.5 ppm throughout the country. The project is denoted as Austrian Geoid 2000 to indicate that the resulting product is intended to survive the turn of the century.

The new computation of the Austrian geoid will be performed by three approaches, (1) the conventional least squares collocation method, (2) the fast collocation method which implies gridded input data and a symmetric block Toeplitz matrix for the covariance function, and (3) the gravimetric solution by the Fast Fourier Transform based on either a planar approximation or a spherical approach for the kernel functions.

As far as Austria is concerned, the data input consists of a 50 × 50 m digital terrain model, some 30.000 gravity data, about 700 deflections of the vertical, and GPS derived points. From the neighboring countries, gravity and height information is available in different quality and density.

1. Least squares collocation today

Slightly more than a quarter of a century ago, the estimation of linear functionals of the anomalous potential based on heterogeneous and noisy gravity data, one of the key problems in physical geodesy, was not yet solved. The mathematical solution of this problem was given by [6] and extensively elaborated by [8] and other scientists and is known as "least squares collocation" (LSC).

The theoretical beauty of LSC has one practical drawback: the processing quickly exceeds the computational capacity of the computer because the solution time increases with the third power of the size of the data set. Therefore, numerous efforts have been made in tuning LSC to manage large data sets.

Among the various methods, the following techniques have been applied frequently and are capable of reducing the LSC computational effort:

a) The LSC patchwork method. The area under consideration is subdivided into a number of overlapping subareas. For each subarea, the LSC solution is performed. The solution for the whole area is obtained by "glueing" together (in a mathematical sense) the subarea solutions.
b) LSC with finite covariance functions. The correlation of data decreases with the separation of the data points. Data with a large spatial separation are almost uncorrelated. Therefore, using a covariance function with finite support produces a band-structured covariance matrix which significantly reduces the computational effort for LSC processing of large areas.
c) Local LSC solutions. Data interpolation and differentiation are mainly affected by the local data environment; the effect of remote data is often negligible. Based on this principle, local LSC solutions with only a small data set can be obtained very efficiently by updating the inverse covariance matrix and the solution vector when the prediction point is moved over the prediction area.
d) Fast Fourier Transform (FFT) solution. For planar gridded homogeneous data sets with homogeneous noise and a covariance function depending on the planar distance, the covariance matrix is a block Toeplitz matrix consisting of symmetric Toeplitz blocks. This specific situation offers the transformation of the LSC solution into the frequency domain by means of the FFT algorithms, cf. Eren [2]. However, errors due to edge effects caused by the finite grid must be carefully considered.
e) Fast collocation. For homogeneous data with homogeneous data noise on a geographical grid and a covariance function depending on the spherical distance, the covariance matrix, due to the convergence of meridians, has no longer the block Toeplitz structure of symmetric Toeplitz blocks. The symmetric Toeplitz structure of each block is preserved, but the block Toeplitz structure is lost. This fact can be overcome as outlined below (Sect. 2.2).

2. The Austrian Geoid 2000

Due to the steadily increasing accuracy requirements, a new effort will be made to recompute the Austrian geoid. For reasons of comparison, three groups will compute independently three different methods: (1) the conventional least squares collocation, (2) the fast collocation, and (3) a gravimetric solution by using the FFT. Some brief explanations of the typical characteristics of these methods are given.

2.1. Conventional least squares collocation

The main input source for the geoid used so far in Austria are deflections of the vertical. The new solution will use heterogeneous data, i.e., gravity data and deflections of the vertical. In addition, GPS data will be used to account for the datum problem.

Details on the solution as realized in the GRAVSOFT program package are given by [14]. This program uses stepwise least squares collocation. The method requires data sets with known standard deviations and, in addition, isotropic covariance functions being specified by a set of empirical anomaly degree-variances. For the input of observations, the GRAVSOFT program package allows potential coefficients, mean or point gravity anomalies, height anomalies, deflections of the vertical, gravity gradients, and density contrasts.

2.2 Fast collocation

Among the previously described methods, the favorite candidate is fast collocation because it is both extremely efficient and provides at the same time an exact solution on the sphere (in contrast to the planar FFT approach). The idea of fast collocation is simple: for a small area on the sphere, a planar grid may be used as a good approximation for a geographical grid. As a consequence, the block Toeplitz structure of the covariance matrix for the planar case may be used as a good approximation for the non-block Toeplitz structure of the covariance matrix for the geographical case.

Following [1], the covariance matrix C may be split into

$$C = \tilde{C} + \delta C \qquad (1)$$

where $\tilde{C}$ represents the block Toeplitz matrix of symmetric Toeplitz blocks, and the matrix δC accounts for the deviation of the spherical from the planar case. The diagonal elements of each block correspond to the covariances between data on the same meridian. Therefore, δC has zeroes on each block diagonal. The size of the off-diagonal elements in each block depends on the grid size and can reach about 10% of the diagonal elements of C for solutions such as the one considered here.

This small deviation suggests the application of an iterative solution with $\tilde{C}$ as the zero-order approximation of the covariance matrix. Denoting the data vector by y and the solution vector by x, the iterative solution is accomplished by

$$\tilde{C} x_{n+1} = y - \delta C x_n. \tag{2}$$

It is important to note that the product $\delta C x_n$ can be computed very rapidly if advantage of the structure of δC is taken: by properly arranging the elements of δC in a vector, the product $\delta C x_n$ can be transformed into a circulant convolution of two vectors which can be computed very efficiently by the Fast Hartley Transform (FHT) by taking into account the convolution theorem.

The convergence rate of Eq. (2) can be improved dramatically by a skillful preconditioning. Two conflicting requirements must be fulfilled by a preconditioner: first, it should be as simple as possible, and, second, it should be as close as possible to the inverse of the operator. The second requirement is certainly achieved by $\tilde{C}^{-1}$ as preconditioner. Therefore, the proposed collocation solution for the Austrian geoid project will focus on a preconditioned conjugate gradient method with incorporated FHT.

The proposed LSC solution will be supplemented by the usual data reduction due to residual terrain and a high resolution geopotential model. Gridded residual gravity data for Austria and all neighboring countries, at least 100 km beyond the Austrian border, will be used.

According to feasibility studies which have been conducted recently, a relative geoid accuracy of about 0.2 – 0.3 ppm may be expected for the entire country.

2.3. Gravimetric geoid by FFT techniques

The classical formula to determine the geoid from gravity data is the Stokes formula

$$N = \frac{R}{4\pi\gamma} \iint_\sigma \Delta g \, S(\psi) \, d\sigma \tag{3}$$

where N denotes the geoidal undulation, R is the radius of a sphere, γ is a mean value of gravity, σ indicates the unit sphere, Δg are the gravity anomalies, and $S(\psi)$ is the Stokes function. The gravity anomalies Δg refer to the geoid. Thus, measured surface gravity data must be reduced to the geoid by a terrain reduction using height data (digital terrain model) and further reduced by the global geopotential model. The reduced data is used to generate the residual part of the geoid by means of (3). The final geoidal undulation results from the residual part, the reference undulation computed by the geopotential model, and the indirect effect (which may be derived from the height data).

The solution of the gravimetric method may be carried out conventionally (e.g., by numerical integration) or by the FFT technique. Several approaches for the FFT were developed: the planar approximation, see [10], the spherical approach, see [11], and other methods.

The principle of the planar approximation is expressed by the following equation

$$N(x_p, y_p) = \frac{1}{2\gamma} \iint_E \frac{\Delta g(x, y)}{\sqrt{(x_p - x)^2 + (y_p - y)^2}} \, dx \, dy \tag{4}$$

where the geoidal height at x_p, y_p is computed from Δg in an area E. This approximation is now a two-dimensional convolution integral. The application of a two-dimensional FFT is straightforward. The error inherent in the planar approximation will grow with the integration area.

The drawback of the planar approximation may be circumvented by the spherical approach where the Stokes integral is transformed to a two-dimensional convolution integral by a modification of the Stokes function. The evaluation is performed on the sphere which causes the superiority compared to the planar approximation. However, [11] introduces also an approximation by using a mean latitude for each grid mesh. The geoid undulations for all grid points can simultaneously be computed by applying a two-dimensional FFT accordingly.

The Stokes function may also be expressed as a convolution in east-west direction (along a parallel), because the Stokes function is constant for all computation points on one parallel, cf. [7]. Applying a one-dimensional FFT, the simultaneous computation of geoid undulations on a parallel is possible without approximation as far as the Stokes function is concerned. This approach was proposed by [5].

Detailed formulas of these and other approaches may be found e.g. in [7] and in [9].

2.4. Available data sets

Gravity data

For Austria, some 32.400 gravity data is available at the Section of Physical Geodesy of the

Technical University Graz. This data was provided from several institutions: Institute of Meteorology and Geophysics of the University Vienna, Institute of Geophysics of the University Leoben, Austrian Petroleum Industry, Institute of Geophysics of the Technical University Clausthal, and the Federal Office of Metrology and Surveying. The data refer to the Austrian gravity network which is compatible with the international system IGSN71. The position parameters referring to the gravity data are related to the datum of the former Military Geographical Institute (MGI), i.e., a local datum associated with the Bessel ellipsoid. Using a grid with a mesh size of 2 x 2 km, the gravity data set may be reduced to 14.255 data for Austria.

Deflections of the vertical

At 683 homogeneously distributed points in Austria, deflections of the vertical are available. The data refers to the same local datum as the gravity data (datum of MGI associated with the Bessel ellipsoid). This data was the primary source of the previously computed Austrian astrogeodetic geoids, cf. [3], [4], [12], [13].

GPS data

From several campaigns, GPS data is ready to be used all over Austria. The main purpose of the introduction of GPS data is the possibility of an accurate datum relation. The GPS data refers to a geocentric system, e.g., the World Geodetic System 1984 (WGS-84). The most important Austrian campaigns since 1990 are the "Austrian Geodynamic Reference" campaigns AGREF90, AGREF92, and AGREF94. Some 75 GPS points established by these campaigns are located on Austrian territory.

Digital terrain model

The Federal Office of Metrology and Surveying provides a high resolution 50 x 50 m digital terrain model for Austria. The positions of the grid points are expressed in geographical coordinates φ, λ. The heights refer to the official Austrian height system consisting of normalorthometric heights associated with the datum point Molo Sartorio in Trieste, Italy.

Surface density model

The two-dimensional surface density model, cf. [15], was derived from a geological map of Austria comprising 40 regions and twelve different densities between 2000 and 2850 kg/m^3.

Global earth model

For the low to medium frequency part of the gravity field of the earth, a global geopotential model (e.g., OSU81 or the model being currently developed by DMA) will be used.

Data of neighboring countries

Gravity and DTM data of all neighboring countries, i.e., Germany (Bavaria and Baden-Württemberg), Czech Republic, Slovakia, Hungary, Slovenia, Italy, Switzerland, are available. The gridding of the data is different. However, all data will be transformed to mean values in a 3′ x 3′ grid. The densification of data is performed by prediction and interpolation, thinning is achieved by averaging data.

3. Conclusion

The three approaches, (1) the conventional least squares collocation method, (2) the fast collocation method, and (3) the gravimetric solution by the FFT will be computed independently from three different groups. After a comparison of the results, the Austrian geoid 2000 will be established. It is almost unnecessary to say that the fast collocation method will yield this Austrian geoid 2000. The reasons are the computational efficiency compared to the conventional least squares collocation method and the superiority with respect to input data compared to the purely gravimetric solution.

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The Austrian Geodynamic Reference Frame (AGREF) Motivation and Results

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Abstract

A summary of the works on AGREF is presented and a review of the accuracy of the results is given. Some prospects of future related activities are mentioned.

Zusammenfassung

Die Arbeiten an AGREF werden zusammengefaßt, die Resultate in Hinblick auf ihre Genauigkeit durchleuchtet und die Zukunftsaussichten betrachtet.

1. Preliminary Remarks

This contribution presents the complementary written summary to a poster presented at the IUGG XXI General Assembly, Boulder, July 2–14, 1995. A special monograph which will contain details of the AGREF activities, including the final coordinates and station documentations, will be published in the course of 1996.

2. Objectives

The objectives remained the same as mentioned in [1], namely to establish a 3D homogeneous reference frame with a total r.m.s. of better than ± 1.5 cm, to support the Austrian Geoid at the cm-level, to monitor regional crustal movements, and to link national and international networks.

In future AGREF may also be used for further objectives, like to provide base stations for DGPS and other real time applications.

3. Realization

3.1 Concept

During the last years the accuracy of GPS-coordinates derived from continuous observation periods of some days could be improved in such a way that it competes with SLR/VLBI methods, without however replacing them for