Paper-ID: VGI_198001

A New Determination of the Height of the World's Highest Peak

Jing-Yung Chen [1], Dan-Sun Gun [2]

[1] *Research Institute of Geodesy and Photogrammetry, Peking, China, derzeit Institut für Landesvermessung und Photogrammetrie, Technische Universität Graz, Rechbauerstraße 12, A-8010 Graz*
[2] *Research Institute of Geodesy and Photogrammetry, Peking, China*

Österreichische Zeitschrift für Vermessungswesen und Photogrammetrie **68** (1), S. 1–19

1980

BibTeX:

```
@ARTICLE{Chen_VGI_198001,
Title = {A New Determination of the Height of the World's Highest Peak},
Author = {Chen, Jing-Yung and Gun, Dan-Sun},
Journal = {{\"O}sterreichische Zeitschrift f{\"u}r Vermessungswesen und
   Photogrammetrie},
Pages = {1--19},
Number = {1},
Year = {1980},
Volume = {68}
}
```

A New Determination of the Height of the World's Highest Peak

Von *Jing-Yung Chen*[1]), *Dan-Sun Gun,* Peking, China

Vorbemerkung

Die VR China hat 1975 eine wissenschaftliche Expedition auf den höchsten Berg der Welt durchgeführt, bei welcher erstmals auch geodätische Ziele verfolgt wurden. Um die genaue Höhe des Quomolangma Feng genannten Mount Everest zu bestimmen, wurden auf diesem ein 3,5 m hohes, trigonometrisches Signal errichtet und von umliegenden, etwa 6.000 m hohen Stationen trigonometrische Messungen ausgeführt. Die Höhen und Positionen dieser Stationen wurden durch Nivellements, Schweremessungen und trigonometrische Messungen ermittelt und an das geodätische Grundsystem der VR China angeschlossen.

An der Ausarbeitung dieser außerordentlichen geodätischen und bergsteigerischen Leistung hat Herr *Jing-Yung Chen,* wissenschaftlicher Mitarbeiter des Forschungsinstitutes für Geodäsie und Photogrammetrie des chinesischen Nationalbüros für Geodäsie und Kartographie in Peking und der Verfasser des folgenden Beitrags, maßgebend mitgearbeitet.

Herr Jing-Yung Chen hat sein Studium 1960 in Wuhan abgeschlossen, war bis 1974 Forschungsassistent und Lehrer in der Abteilung für Astronomische Geodäsie und ist seit dieser Zeit wissenschaftlicher Mitarbeiter im Forschungsinstitut für Geodäsie und Photogrammetrie in Peking. Herr J.-Y. Chen beschäftigt sich mit wissenschaftlichen Problemen der Vermessungsgrundlagen in der VR China und hat dazu eine Reihe von Beiträgen publiziert. Seit Beginn 1979 absolviert er an der TU in Graz einen zweijährigen Studienaufenthalt mit dem Ziel, westliche geodätische Verfahren näher kennenzulernen und Vorschläge für eine Verbesserung der chinesischen Vermessungsgrundlagen zu erarbeiten. Der Aufenthalt von J.-Y. Chen in Graz ist ein erster Schritt zu einer vom Unterzeichneten durch Besuche in der VR China angestrebten wissenschaftlichen Zusammenarbeit auf dem Gebiet der Geodäsie zwischen der VR China und Österreich.

Karl Rinner, Graz

Abstract

The Quomolangma Feng[2]) (Mt. Jolmo Lungma) is the world's highest peak. Its accurate height above mean sea-level has long been the concern of surveyors and geographers the world over. In 1975, members of a Chinese mountaineering expedition for the first time erected on the summit of the QF[3]) a metallic target and also measured there the thickness of the covering snow. At the same time, and in coordination with the expedition, a surveying group carried out levelling, triangulation, astronomical and gravimetric measurements within a close range of the QF. The nearest gravity station was at an elevation of 7790 m and at a distance of only 1.9 km from the summit. In this paper the computation for determining the height of the peak according to rigorous geodetic theory is described in detail, and some questions regarding atmospheric refraction, deviation of the vertical, gravity and the geoid are discussed.

Most of the short-comings have now been overcome which existed in the previous height determinations. The height of the QF as obtained in 1975 is 8848.13 m ± 0.35 m above mean sea-

---

[1]) Presently at Institut für Landesvermessung und Photogrammetrie, Technische Universität Graz, Österreich.

[2]) It was called Mount Everest before.

[3]) QF is used as an abbreviation of "Quomolangma Feng" in this paper.

level. This may be the most accurate value that has so far been obtained for the highest peak of the world.

Introduction

The Quomolangma Feng is the world's highest peak. It is situated on the border between China and Nepal. Its precise height has long been a matter of interest among the surveyors and geographers the world over [4], [7], [8]. In 1975, a Chinese mountaineering expedition erected for the first time a metallic target on top of the peak (Fig. 1), and measured the thickness of the covering snow at the spot. At the same time a Chinese surveying group carried out coordinated geodetic measurements within a close range of the QF (about 30 km × 30 km).

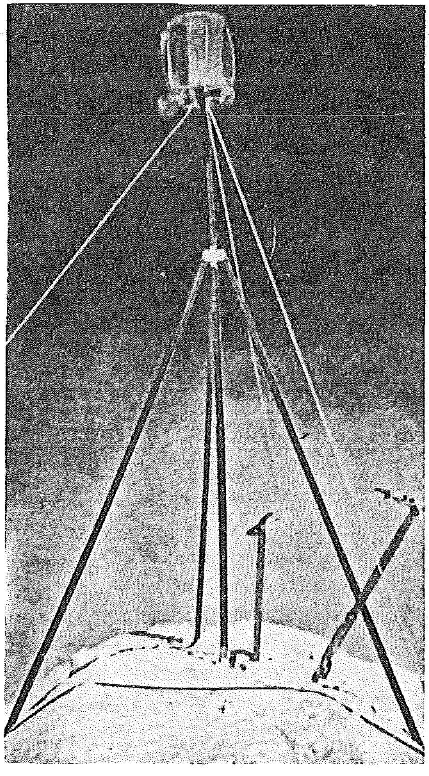
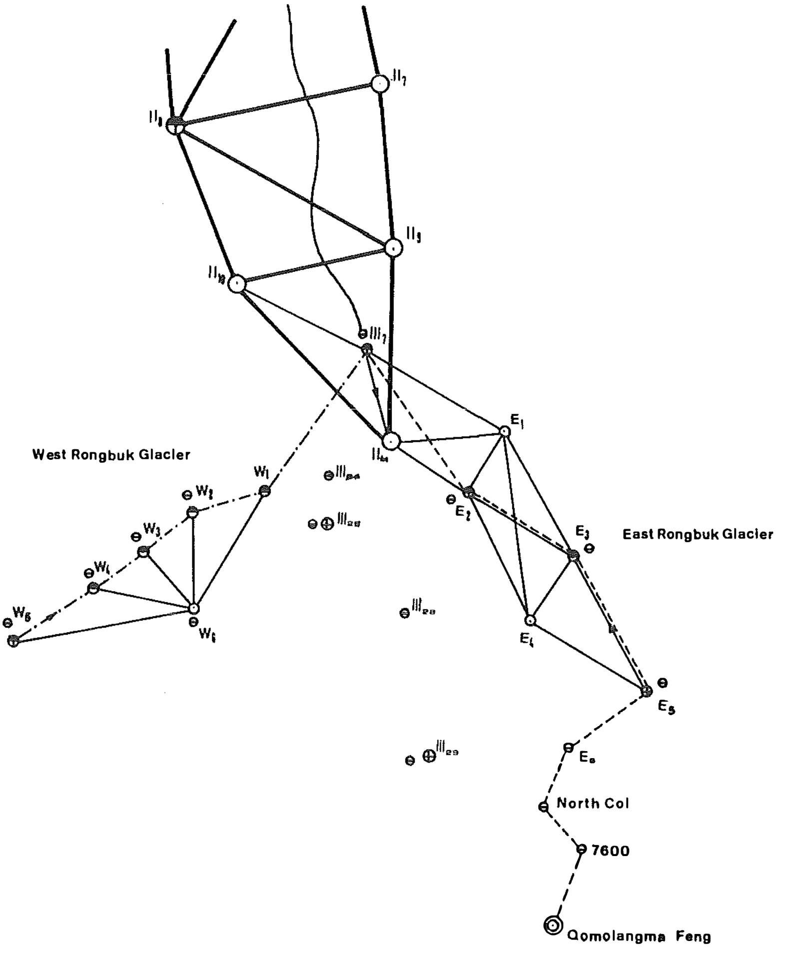


Figure 1

As a result of these efforts, it became possible to compute the height of the QF by rigorous geodetic methods. The height value (8848.13 m ± 0.35 m) must be considered the most accurate one thus far obtained [5] [17].



◯ second-order station	—— third-order side
o third-order station	—·— EDM side
⊖ gravimetric station	—➤— astronomical azimuth
⊕ astronomical station	- - - astronomical levelling line
▬ second-order side	∿ levelling line
● stations occupied for intersecting the QF	

The determination of the QF in 1975.

Figure 2

1. Field Work

Beginning from the national 1st order traverse at Dingri, in the south of Tibet, a 2nd order triangulation chain about 60 km long was laid out toward the direction of the QF. A 3rd order chain along the East Rongbuk Glacier, and an EDM traverse along the West Rongbuk Glacier, were tied to the 2nd order stations, and controlled by Laplace azimuths at their end points (Fig. 2). The standard error (s. e.) of angle measurement of these 3rd order stations after adjustment is $\pm$ 1.″73. Among the 3rd order stations were nine from where the target was visible; these were used for intersecting the QF. They were 8.5 km to 21.2 km away from the peak (12.8 km on the average); elevations ranged from 5600 m to 6240 m (5784 m on the average). The maximum angle of intersection was 69°.

Astronomical latitude and longitude were determined on 15 stations in this surveyed region, their s. e. are $\pm 1''$ and $\pm 0.^{s}06$ respectively. Among these astronomical stations the nearest one to the QF, III 29, is at an elevation of 6336 m and only 5 km away from the peak. In addition, five Laplace azimuths were determined with a s. e. of $\pm 2''$.

Gravimetric measurements were made on 20 points in this region, their s. e. is $\pm$3 mgal. The two highest points are at elevations of 7050 m and 7790 m, respectively, the latter being only 1.9 km apart from the summit. They are the highest gravity points on land that have ever been measured in the history of geodetic surveying.

Starting from a national benchmark (BM) at Dingri, a 2nd order levelling line was measured southward up to a BM south of the Rongbuk Lamasery. The triangulation station III 7, about 5683 m above mean sea-level and 13.6 km north of the summit, was connected to this BM by means of spirit levelling; the mean square value of random errors of mean height difference per km is $\pm$0.71 mm. The height of III 7 was used as the basis for the succeeding height determination.

Reciprocal trigonometric levelling was carried out between the 3rd order stations to determine their own height. As soon as the metallic target was erected on the summit, zenith distances to the target were observed from all the nine stations during three consecutive days.

In the meantime, a number of sounding balloons were lifted to measure the temperature gradient of the upper atmosphere; the values were used later to determine a reliable coefficient of refraction.

The height of the target and the thickness of the covering snow at the spot were measured with 3.51 m and 0.92 m, respectively.

2. *Height Computation*

Restrictions due to the specific geographical conditions in the area led to the following general procedure for determining the height of the QF:

- Extension of the national geodetic and levelling nets to a range as close to the QF as possible;
- Observation of the horizontal angles and zenith distances to the peak from a number of selected triangulation and traverse stations, so as to determine the geographical coordinates of the QF and the height differences between the observation stations and the summit;
- Finally reduction of the measured height to mean sea level (orthometric height) utilizing the data of the astronomical and gravimetric measurements in the surveyed region.

The topography around the QF is very rough. The geoid in this region is deemed to be rather complicated and cannot be considered as being close to a reference ellipsoid. Consequently, the general principle in a rigorous computation has to be that, the surface of an ellipsoid is taken as the basic reference surface to which all the observed data must be reduced [2], [14]; then the horizontal and height computations are carried out.

The principal steps of data processing in this computation are as follows:

2.1 The Normal Height of Station III 7, $H^{\gamma}_{III\,7}$

Station III 7 is connected with the national datum 0 at Tsingtao through the national 1st and 2nd order levelling net. In order to obtain $H^{\gamma}_{III\,7}$, two correction terms should be added to the levelled height difference, $\sum\limits_{0}^{III\,7} \Delta h$, from 0 to III 7, by the following formula [11], [16]

$$H^{\gamma}_{III\,7} = \sum_{0}^{III\,7} \Delta h + \sum_{0}^{III\,7} \varepsilon + \sum_{0}^{III\,7} \frac{g - \gamma}{\gamma} \Delta h \qquad (1),$$

where ε is the correction caused by the non-parallelism of the level surfaces in the normal gravity field; g and γ are values of measured gravity and normal gravity along the line of levelling, respectively. The s. e. of $H^{\gamma}_{III\,7}$, M_1, is estimated to be

$$M_1 = \pm\, 0.14 \text{ m} \qquad (2),$$

according to the usual accuracy estimation formula for spirit levelling in a normal height system [16].

2.2 Geodetic Height of III 7, $H^{e}_{III\,7}$

The geodetic height H^e_i is the summation of normal height H^{γ}_i and height anomaly ζ_i for any station i. As regards III 7, we have

$$H^{e}_{III\,7} = H^{\gamma}_{III\,7} + \zeta_{III\,7} \qquad (3).$$

In the process of computing the height of the QF, $\zeta_{III\,7}$ will be cancelled as shown in equation (14) below, hence the error in $\zeta_{III\,7}$ has no effect on the results, and an approximate value of it will be sufficient.

2.3 Geodetic Heights of the Triangulation and Traverse Stations, H_i^e

Corrections for the deviation of the vertical are applied to the observed value of the zenith distance Z, reducing it to the geodetic zenith distance Z^e referred to the ellipsoid [11]:

$$Z^e = Z + u, \; u = \xi \cos A + \eta \sin A \qquad (4),$$

where u is the component of the astrogeodetic deviation of the vertical in the observed direction, the geodetic azimuth of which is A, and ξ, η are the components of the deflections of the vertical.

Then the common formula of reciprocal trigonometric levelling is used to compute the differences of geodetic heights, $\Delta h_{III\,7\,i}$ between III 7 and any 3rd order triangulation and traverse station i.

The geodetic height for station i can now be written as

$$H_i^e = H_{III\,7}^e + \Delta h_{III\,7\,i} \qquad (5),$$

2.4 Geodetic Height of the QF, H_Q^e

The zenith distances to the target on the peak observed from station i are corrected with Equ. (4). The difference of geodetic height between the QF and station i, Δh_{iQ}, is computed according to the following precise formula [12]

$$\Delta h_{iQ} = s\,(1 + \frac{H_Q^e}{R})\,\mathrm{ctg}\, Z^e + (\Delta_H + \Delta_K)\, s^2 + l - a \qquad (6),$$

where

$$\Delta_H = \frac{H_Q^e}{3\,R^2} + \frac{H_i^e}{6\,R^2};$$

$$\Delta_K = \frac{1}{2\,R}\,(1 - \frac{k}{\sin Z^e});$$

"l" is the height of the instrument at an observed station; "a" is the height of the target at the QF; "s" is the distance on the ellipsoid from station i to the QK; "R" is the radius of curvature of the normal section of the ellipsoid along the line of observation; "k" is the coefficient of refraction.

The geodetic height of the QF is then

$$H_{Qi}^e = H_i^e + \Delta h_{iQ} \qquad (7).$$

The results obtained from each station are presented in Table 1.

Assigning to the result of each station a weight inversely proportional to the square of its distance to the peak, the weighted mean of the geodetic height of the QF, H^e_Q, is found.

The maximum difference between H^e_{Qi} derived from various stations is 1.4 m. M_2 is the s. e. estimated from the residuals:

$$M_2 = \pm 0.18 \text{ m} \tag{8}$$

M_2 contains the errors in $\Delta h_{III\,7\,i}$ and Δh_{iQ}, but does not include the error of the $H^\gamma_{III\,7}$, therefore it really presents a measure for the error of trigonometric levelling.

Table 1 The Height Values of the QF from Different Stations

Observation Station i	Distance s (km)[1]	Height of Station H^γ_i (m)[1]	Coefficient of Refraction k	Normal Height of the QF H^γ_{Qi} (m)	Residual (m)
II 8	21.2	6120	0.0744	8846.21	− 0.64
III 7	13.6	5683	0.0810	8846.59	− 0.26
East 2	10.0	6242	0.0758	8846.88	+ 0.03
East 3	8.5	6168	0.0761	8846.31	− 0.54
West 1	11.3	5602	0.0793	8846.39	− 0.46
West 2	11.7	5748	0.0772	8847.36	+ 0.51
West 3	12.0	5750	0.0828	8847.61	+ 0.76
West 4	12.5	5772	0.0804	8847.63	+ 0.78
West 5	14.7	5798	0.0820	8846.85	0.00
Mean	12.8	5874	0.0788	8846.85[2]	

[1]) only approximate values are given here [2]) weighted mean

2.5 Normal Height of the QF, H^γ_Q

According to (3), we have

$$\left.\begin{aligned} H^\gamma_Q &= H^e_Q - \zeta_Q \\ &\text{or} \\ H^\gamma_Q &= H^e_Q - \zeta_{III\,7} - (\zeta_Q - \zeta_{III\,7}) \end{aligned}\right\} \tag{9}$$

where the difference of the height anomaly from III 7 to the peak $(\zeta_Q - \zeta_{III\,7})$, can be computed by the following precise formula of astronomical levelling [16], [12]

$$\left.\begin{aligned} \zeta_Q - \zeta_{III\,7} &= \sum_{III\,7}^{Q} u\, s - \sum_{III\,7}^{Q} \Delta E \\ \Delta E &= \frac{g - \gamma}{\gamma} \Delta H \end{aligned}\right\} \tag{10}$$

The second term of equation (10) relates to gravity anomalies. When the difference of height is great, then the value of this term always surpasses that of the first term, relating to the deviation (compare Table 2). Hence, in general speaking, the second term should not be neglected in mountain regions. From the last column of Table 2 one gets the value of $(\zeta_Q - \zeta_{III\,7})$, as –1.825 m.

Table 2 Computation of Differences of Height Anomalies (unit: meter)

Point Along the Line	s	H	u s[1])	ΔE	u s – ΔE
III 7		5683			
	4279		– 0.012	+ 0.180	– 0.192
East 2		6242			
	2656		– 0.008	– 0.026	+ 0.018
East 3		6168			
	3042		– 0.117	+ 0.047	– 0.164
East 5		6301			
	2819		– 0.091	+ 0.101	– 0.192
East 6		6565			
	1329		– 0.058	+ 0.200	– 0.258
North Col		7050			
	1394		– 0.065	+ 0.321	– 0.386
No. 7600		7790			
	1947		– 0.176	+ 0.475	– 0.651
The QF		8849			
Sum	17466		– 0.527	+ 1.298	– 1.825

[1]) u is the deviation of the vertical of station along the astronomical levelling line
s is the distance between two consecutive stations

The gravity values at all points but the QF along the line of astronomical levelling given in Table 2 were actually measured. The gravity value of point No. 7600, mentioned above, is particularly important for a reliable prediction of gravity value of the QF. The values of the deviation of the vertical at points III 7. East 2 and East 5 were obtained directly by astronomical observations, but those of the other stations are deduced by prediction. The s. e., M_3, of the difference of height anomalies $(\zeta_Q - \zeta_{III\,7})$ is estimated according to the following formula

$$M_3 = \pm \sqrt{\frac{m_u^2}{g^2}\, s_m \sum_{III\,7}^{QF} s + \sum_{III\,7}^{QF} \left(\frac{\Delta H}{\gamma}\, m_g\right)^2} \qquad (11),$$

where m_u and m_g are the s. e. for the value of deviation of the vertical and of the gravity anomaly at the corresponding station (it will be discussed later in detail).

$$M_3 = \pm 0.10 \text{ m} \tag{12}$$

The result obtained for the normal height of the QF from equation (9) is

$$H_Q^\gamma = 8846.85 \text{ m} \tag{13}$$

By substituting equations (7), (5), (3) into (9) successively, it is easy to obtain

$$H_Q^\gamma = H_{III\,7}^\gamma + \Delta h_{III\,7\,i} + \Delta h_{iQ} - (\zeta_Q - \zeta_{III\,7}) \tag{14}$$

It can be seen from this formula that an error in $\zeta_{III\,7}$ will not affect the result of H_Q^γ.

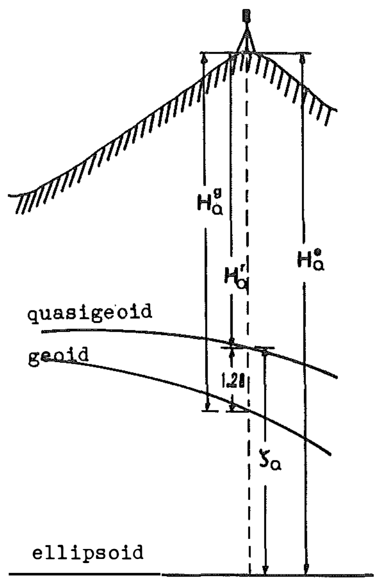
According to equations (14), (12), (8), (2) the s. e., M_{HQ}^γ, of the normal height of QF can be written as

$$M_{HQ}^\gamma = \sqrt{M_1^2 + M_2^2 + M_3^2} = \pm 0.25 \text{ m} \tag{15}$$

2.6 The Orthometric Height of the QF, H_Q^g

The relation between normal height and orthometric height of the QF is [11], [16]

$$H_Q^g = \frac{\gamma_m}{g_m} H^\gamma_Q \tag{16},$$



The relation of the basic surfaces at the QF.

Figure 3

where γ_m, the mean normal gravity, can be obtained by simple computation. The method for computing the mean actual gravity g_m will be outlined below. By formula (16), the difference between the orthometric height and the normal height of QF is computed to be 1.28 m. That means the quasigeoid under the QF lies 1.28 m above the geoid (Fig. 3).

Consequently the orthometric height of the QF is 8846.85 + 1.28 = 8848.13 m above mean sea-level of the Yellow Sea, or

$$H_Q^g = 8848.13 \text{ m} \qquad (17).$$

The error for computing g_m at QF is about ±28 mgal (referred to 3.6). Its effect on the conversion from H_Q^γ to H_Q^g may be estimated, according to (16), as

$$M_4 = \pm 0.25 \text{ m} \qquad (18).$$

So, the s. e., M_{HQ}^γ, of the orthometric height of the QF determined in 1975 can be estimated as

$$M_{HQ}^\gamma = \pm \sqrt{M_{HQ}^{2\gamma} + M_4^2} = \pm \sqrt{M_1^2 + M_2^2 + M_3^2 + M_4^2} \qquad (19).$$
$$= \pm 0.35 \text{ m}$$

3. Discussion

3.1 The Target

The height of the target was designed in accordance with the topography on the summit as described by Chinese mountaineers who scaled the park in 1960, and verified by aerial photos of the QF area. Estimates based on the elevations and positions of the observation stations relative to the peak showed that a 3.5 meter-high target on the summit would be visible from all the observation stations. The peak is always covered by white snow and the background for observations is a bright blue sky. Many practical tests were needed to find that the target must be painted in red to provide good contrast for accurate pointing. Besides, the target had to be made of light alloy with high strength, weighing only 3.74 kg, with flexible legs and folding cylinder for convient carrying during climbing. Yet its structure was designed to withstand strong gales always prevailing on the summit. The central rod of the target is graduated in centimeters, so that an accurate reading for the height of the target can easily be taken. When erected, the target was secured with three nylon ropes fastened to ice picks firmly anchored into the icy ground. As a matter of fact, it took less than half an hour to erect it on the peak securely, and in spite of the storms and gales prevailing on the peak, the target has remained standing for at least more than 3 years, for in May 1978 some Austrian mountaineers took a summit photo showing themselves together with this target (Fig. 6). Practice has thus proved that such a target is fit for extreme conditions and would be worth introducing elsewhere.

3.2 Field-data and their accuracy

In the close vinicity of the QF area besides the performance of geodetic measurements a lot of astronomical and gravimetric data have been gained. Some features of the data and their accuracy are listed in Table 3 and 4.

Table 3 Features of the Field-data in the QF surveyed Area

target	height of the target itself on the summit	3.51 m
	depth of the covering snow under the target	0.92 m
astronomical measurement	height of the highest BM (2nd order), II 8	6120 m
	distance of the nearest BM (4th order) to the QF, III 7	13.6 km
	height of the highest astronomic station, III 29	6336 m
	distance of the nearest astronomic station to the QF, III 29	5 km
	sum of the astronomic stations	15
	average distance between two consecutive astronomic stations along the astronomical levelling line	2.5 km
gravimetric measurement	height of the highest gravimetric point, No. 7600	7790 m
	distance of the nearest gravimetric point to the QF, No. 7600	1.9 km
	sum of the gravimetric points	20
stations for intersecting the QF	height of the highest station	6240 m
	average height	5784 m
	the shortest distance to the QF	8.5 km
	average distance to the QF	12.8 km
	sum of stations	9
	maximum angle of intersection	69
	days lasted for intersection	3 days
sounding balloon measurement	days lasted for sounding balloon measurement	9 days

Table 4 Standard Errors of the Field-data

mean height difference for one km spirit levelling from BM to III 7	±0.71 mm
mean height difference in reciprocal trigonometric levelling for one side of a triangle	±0.04 m
astronomical latitude (Talcott)	$\pm 1''.0$
astronomical longitude (Tsinger)	$\pm 0^{s}.06$
astronomical azimuth (Hour Angle of Polaris)	$\pm 2''.0$
angle measurement after adjustment	$\pm 1''.8$
distance measurement	±10 ppm
gravimetric measurement	±3 mgal

Fig. 6 The target on the summit of the QF
(photo by Austrian mountaineer Robert Schauer, May 9, 1978)

3.3 The Vertical Refraction of the Atmosphere

In order to understand correctly the rules concerning the vertical refraction of the atmosphere in the QF area, many experiments were carried out on "the roof of the world" before the height determination. One kind of experiment was the reciprocal trigonometric levelling on some stations (connected by the spirit levelling line) under different meteorological conditions. The following results were obtained for the refraction coefficient in different situations:

1. The value k of the coefficient of refraction is smallest and most stable between 12^h–15^h (local time), consequently this is the most favourable time for observing zenith distances.
2. When the trigonometric levelling was carried out in the triangulation and traverse, the mean value of the refraction coefficient for the time from 12^h–15^h was about 0.10.
3. The amplitude of diurnal variation of the coefficient of refraction along the sides of triangulation lines has a maximum value of 0.20 and a mean value of 0.07.

Another kind of experiment was carriad out by intersecting the summit of the QF from some triangulation stations, knowing their heights. The aim of this experiment was to verify that the following formula [3] is suitable to calculate the refraction coefficient for sights to the QF:

$$k = 6.706 \frac{P}{T^2} (3.42 + \tau) [1 - \frac{1}{3}(3.42 + \tau)\frac{\Delta h}{T}] \sin Z \qquad (20),$$

where T is the temperature of the air at a station in °K; P is the atmospheric pressure at the observation station in mm of a mercury column; τ is the mean vertical gradient of the air temperature in °C/100 m; Δh is the difference of the height in 100 m; Z is the zenith distance of the line of sight.

Formula (20) is derived according to the physical properties of a free atmosphere and by assuming the vertical gradient τ of the air temperature as constant; in theory it is therefore especially suitable for sights to the QF for the path of ray of such sights always lies high above the ground.

By using the value of k calculated according to formula (20), the height of the QF derived from experimental stations located 8 to 77 km away from the peak showed no apparent systematic difference relating to the length of sight. Besides, for these sights, the amplitude of diurnal variation of the coefficient of refraction obtained by this formula is always less than 0.03, with an average of only 0.017. This is about one quarter of the amplitude for an ordinary terrestrial sight mentioned above. The stability of the refraction coefficient for sights to the QF really corresponds to the practical situation in the area of QF. These results demonstrate the adaptability of the formula for the sights to the QF. Consequently formula (20) was employed to compute the values of k for the sights to the QF in 1975; the values of k were around 0.08 (Tab. 1).

In order to get an accurate value of k it is essential that the vertical gradient of air temperature, τ, must be determined correctly. Some of the following experiences may be useful in this context:

1. τ must be measured in situ practically by radio sounding of the meteorological conditions at different altitude of the atmosphere.
2. It is important to identify the lowest altitude from which the vertical gradient of air temperature begins to enter in the computation of τ. Our experiments showed that the lowest altitude is 500 m above the ground in the QF area.
3. In formula (20) τ is a mean value for the meteorological period in which the temperature, pressure, etc., of the atmosphere are similar. If the observation take an extended time, then one or more different values of τ should be used to compute k for this observation, depending on the number of the local meteorological periods.

3.4 The Prediction of the Deviation of the Vertical

In points without astronomical observations the values of the astrogeodetic deviation of the vertical were predicted on the basis of the known astrogeodetic deviations of the vertical by way of topographic isostatic deviations of the vertical [11], [15]. In computing the topographic isostatic deviations the outmost radius of integration was 670 km and the depth of isostatic compensation was 113.7 km [6], [9], [10].

Some trial computations were made to check the real accuracy of prediction by this method. In one such computation using a total of 15 astronomical stations in the surveyed region, 8 stations were arbitrarily chosen as control points, and the values for the remaining, 7 stations, were predicted. The differences between the predicted and the observed values are listed in Table 5. The s. e. of the prediction as computed from these differences is $\pm 2\overset{''}{.}8$.

At four of the nine 3rd order stations that were used for the observation of the QF, the values of ξ and η were derived directly from astronomical observations, while in the other five stations they were predicted as described above. With an average distance of 11.5 km from the station to the QF, the effect of the prediction error in one station on the height of the QF is estimated to be ± 0.16 m. After averaging the results from all stations, the effect on the mean height of the QF will be not more than ± 0.06 m. An estimate based on equation (11) shows that the error of the predicted deviation of the vertical effects the results of astronomical levelling ($\zeta_Q - \zeta_{III\,7}$) by about $+0.09$ m (here, $m_u = \pm 2\overset{''}{.}8$, $s \Sigma s = 4.36 \times 10^{-7}$). Hence it can be seen that even in such a highly mountainous area, the prediction method for the astrogeodetic deviation of the vertical by way of topographic isostatic deviation of the vertical is justified. The accurracy obtained may not be comparable to that in a flat area, however, its effect on the final results is limited.

Table 5 The Error of the Prediction of the Deviation of the Vertical

Station	v_ξ	v_η
I 2	+ 0."37	+ 3."74
II 3	+ 2.51	+ 2.18
II 4	+ 1.84	+ 2.29
III 8	− 3.94	+ 1.34
III 25	− 3.38	− 1.85
III 29	− 0.56	− 3.84
East 2	− 0.53	+ 2.02
Sum	− 5.25	+ 5.88
[vv]	37.37	48.03
$\pm \sqrt{[vv]/n}$	± 2."31	± 2."62

v: the differences between the prediction and the observed values of the deviation of the vertical at a station.

3.5 Gravity Prediction

We have computed the free-air anomaly, Bouguer anomaly, Bouguer anomaly with terrain correction, and isostatic anomaly[1]) at 20 measured gravity points in the vicinity of the peak. As expected, the variation of the free-air anomalies is largest, while that of the isostatic anomalies is smallest, with an amplitude of variation of only about 40 mgal. Hence the gravity values of the QF was predicted on the basis of the measured gravity value by way of isostatic anomalies [12], [14]. In computing the isostatic anomalies we took 166.7 km as the outmost radius of integration and 113.7 km as the depth of isostatic compensation.

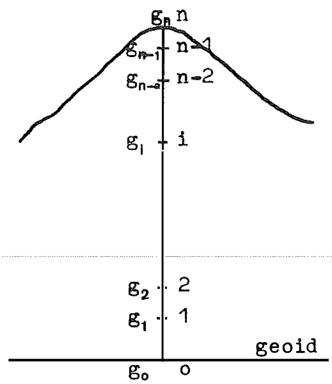
In order to estimate the accuracy of the predicted gravity, we also made a trial computation. The gravity values at two points nearest to the summit, i. e. the North Col (7.050 m) and No. 7600 (7.790 m), which had been measured by gravimeters, were considered as unknown and predicted on the basis of the other measured gravity values. The differences between the predicted and observed values were found to be 15 mgal for North Col and 19 mgal for point No. 7600. Consequently the s. e. for the predicted gravity value at the top of the QF is estimated at ±20 mgal. We also made some tests on selecting a proper depth of isostatic compensation for gravity and deviation prediction in this area. The tests showed, that the accuracy of prediction would deteriorate, if the depth was less than 50 km or more than 200 km.

---

[1]) Obtained after Bouguer. Terrain and isostatic corrections.

3.6 The Mean Gravity Value g_m

In order to compute g_m at the QF, the gravity values at the QF from the ground surface to the geoid had to be determined. As it was impossible to measure the gravity underground, it was calculated on the hypothesis of isostasy: the plumbline from the summit to the geoid was evenly divided into n sections, the dividing points being denoted by 0, 1, 2, . . . (n–1), n (Fig. 4).



The plumb line at the QF is divided into n sections.

Figure 4

Let A_i and B_i be the vertical components of gravitational attraction to the ith point on the plumbline due to topography and isostatic compensation, respectively, counting positively downwards; and Δg_i is the free-air reduction from the summit to point i. We use the following expression with second order terms [13]:

$$\Delta g_i = 0.3086\,[(n-i)\frac{H_Q^g}{n}] - 0.72 \times 10^{-7}\,[(n-i)\frac{H_Q^g}{n}]^2 \qquad (21),$$

The gravity value g_i at the ith point is:

$$g_i = g_n - A_n + A_i + B_n - B_i + \Delta g_i \qquad (22),$$

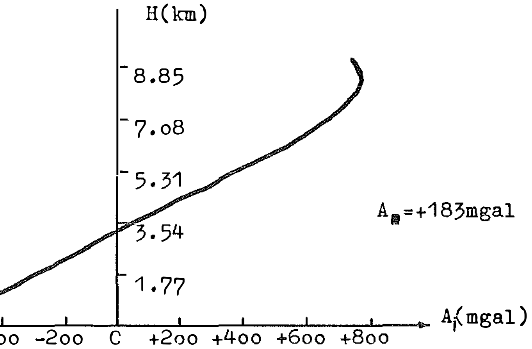
where g_n is the gravity value at the top of the summit. The mean gravity value g_m is then derived by the following formula:

$$g_m = g_n - A_n + A_m + B_n - B_m + 0.1543\,H_Q^g - 0.12 \times 10^{-7}\,H_Q^g\,(2\,H_Q^g + \frac{H_Q^g}{n}) \qquad (23),$$

in which

$$\left.\begin{aligned} A_m &= \frac{1}{n+1}\sum_o^n A_i \\ B_m &= \frac{1}{n+1}\sum_o^n B_i \end{aligned}\right\} \qquad (24).$$

Figure 5



The variation of A_i — the vertical attraction due to topography.

Fig. 5 shows the variation of the attraction due to topography, A_i, with respect to the height of point i. It is seen clearly that there is no linearity in the gravity values along the plumbline under such a high peak. An error of about 100 mgal would be introduced if the calculations were based on linear prediction. According to the trial computation, when $n > 30$, the amplitude of the variation of the mean gravity g_m or the mean normal gravity γ_m was only within 3 to 5 mgal, so we took $n = 50$ for computing g_m and γ_m.

There are two sources of error in calculating the gravity value g_m by the above method: the error of the summit gravity value g_n and that of deducing g_m on the basis of the hypothesis of isostasy. For the value of g_n, a s. e. of about ± 20 mgal due to prediction has been estimated in 3.5. The latter may also be regarded as prediction, however, along the vertical direction on the hypothesis of isostasy. So the same s. e. of ± 20 mgal may well be adapted. By combining the two sources of errors, the s. e. of g_m obtained is estimated to be ± 28 mgal.

3.7 Use of Normal Height as a Medium

The fact that normal height is used as a medium to compute the orthometric height from the geodetic height needs explanation. The geodetic height is the sum of orthometric height and undulation, that means the separation of the geoid from the ellipsoid. In order to determine the undulation, the values of deviation of the vertical on the geoid (not on the earth surface) must be known so that the geoidal undulation could be computed by the simple formula of astronomical levelling. But the deviation on the geoid cannot be obtained directly from astronomical observation unless a correction for the curvature of the plumbline is applied. The curvature of the plumbline is dependent on the mass distribution in the crust of the earth which is not exactly known. Therefore the usual practice is to correct only for the normal curvature of the force line of the normal spheroid, when the elevation is not high. But this simplification would introduce a serious error when the elevation is high, such as in the QF area. Therefore in the computation we have to use normal height as an intermediate result. The unique feature of the normal height system is that it is entirely independent of the mass distribution in the crust of the earth. The normal height can be obtained by applying to the levelled height a correction for gravity anomalies on earth surface. The height anomalies can be obtained from astronomical levelling, using values of ξ and η on the earth surface and supplementing them by gravimetric data. Hence in the process of our computation no assumption is needed on the mass distribution in the earth's crust to obtain the normal height of the peak. The formula used is rigorous, and the accuracy of the result is affected only by the errors of observation and of prediction, but no error exists due to hypothesis. Only in the reduction from the normal height to orthometric height of the QF an assumption on the mass distribution in the earth's crust has to be made. The assumption serves to compute the mean value of gravity g_m along the vertical between the earth surface and the geoid. The error in the last step of the computation is estimated to cause a s. e. of ± 0.25 m of the value of orthometric height of the QF (see 2.6 and 3.6).

4. Conclusion

The new determination of the height of the QF in 1975 has the following specific characteristics:

1. A target was set up on the top of the summit and the thickness of the covering snow on the spot during the observation was measured. This was necessary to reduce an otherwise serious error of sighting the summit and of computing the height of the peak.
2. The normal height system was used as a medium in this computation. As no assumption on the mass distribution in the earth's crust was needed in the steps for computing the normal height of the QF. Therefore the result of this new determination avoided a part of the error caused by the approximation in these assumptions.
3. Sufficient astronomical and gravimetric measurements were made in the close vicinity of the QF area, such that the height of the QF obtained from spirit levelling and trigonometric levelling could be reduced rigorously to the geoid. The resulting height is thus the real orthometric height. The sum of corrections relating to gravity anomalies in our computation amounts to about 2.5 m. That is to say, if computation is made disregarding the gravity data, as it was in the practice of all previous determinations, the resulting error in the computed value of the height of the QF would be more than 2 m. This value itself showes clearly that the field work and the procedure of computation used are both proper and necessary.

The result of this new determination, obtained in 1975, is the orthometric height of the QF with a value of 8848.13 m ± 0.35 m above mean sea-level of the Yellow Sea. This result is claimed to be the most accurate and scientific one that has so far been obtained. Another result of the effort consists of the data processing procedure which proved that it could also be necessfully applied to compute accurately the height of other high mountains.

Acknowledgement

The precise height measurement of the QF could not have been obtained without the exceptional achievements by Chinese mountaineers and the dedication of the accompanying surveying group.

We wish to thank Prof. Dr. Yun-Lin Chen, Chief engineer of the General Bureau of Surveying and Mapping of China, for his valuable suggestions and beneficial instructions during all the course of the work. For the useful and exact calculations we would also like to express our appreciation to our colleagues who took part in this computation.

Finally, it is with gratitude that we acknowledge the support and encouragement given to us by Prof. Dr. Karl Rinner for publishing this paper and for many useful discussions during its preparation. We also wish to thank Dr. Franz Leberl and Mr. Herbert Lichtenegger for their help.

References

[1] *Baeschlin, C. F.:* Lehrbuch der Geodäsie. Zürich (1948).

[2] *Bomford, G.:* Geodesy. Oxford (1971).

[3] *Brocks, K.:* Meteorologische Hilfsmittel für die geodätische Höhenmessung. ZfV ss. 71–76 (1950).

[4] *Bomford, G.:* Geod. Rep. Surv. India (1928–1931).

[5] *Chen, J.-Y.:* The Height Computation of the Quomolangma Feng. Tse-We-Dun-Bao No. 4. Peking (1975).

[6] *Darling, F. W.:* Fundamental Tables for the Deflection of the Vertical. Spec. Publs. U. S. Cst Geod. Surv. No. 243 (1948).

[7] *Graaff-Hunter, J.:* Geod. Rep. Surv. India (1921).

[8] *Guillaume, C. E.:* Tech. Pap. Surv. India (1954).

[9] *Hayford, J. H.:* The Effect of Topography and Isostatic Compensation upon the Intensity of gravity. Spec. Publs. U. S. Cst Geod. Surv. No. 10 (1921).

[10] *Heiskanen, W.:* New Tables for the Reduction of Gravity on the Airy Hypothesis. Helsinki (1938).

[11] *Heiskanen, W., and Moritz, H.:* Physical Geodesy. Freeman (1967).

[12] *Isotov* et al.: The Trigonometric Levelling. Moscow (1958).

[13] *Jing, F.:* Gravimetry. China (1965).

[14] *Jordan, W.* et al: Handbuch der Vermessungskunde. Band 1, 4, 5. Berlin (1965).

[15] *Krasovskij, F. N.:* The Handbook of Geodesy. Vol. 2, b. Moscow (1942).

[16] *Molodensky, M. S.:* Methodes for Study of the External Gravitational Fie AND Figure of the Earth. Moscow (1962).

[17] *Chu, L.:* The Determination of the Height of the Quomolangma Feng. Scientia Sinica Vol. 19, No. 2. Peking (1976).